Approximation of Energy-Optimal Train Control

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15 grudnia 2020

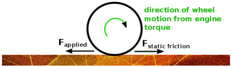
The work was part of the Ecotrain Project "Research and development of a system for optimalisation of railway transport" realised by Business Online Services Sp. z o.o.

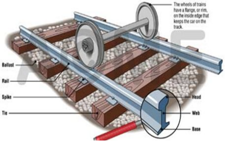
The problem

We consider the mathematical model of the train moving along a segment of the track of the length L between two stops. Let the distance along the track from the start be denoted by x_1. Furthermore, the track is characterized by the following features:

1. the slope $s(x_1)$, represented by the tangent of the inclination angle at the point x_1;

2. the curvature $\kappa(x_1)$ at a given point;
3. the friction coefficient μ_s, corresponding to the static friction steel/steel;





4. the rolling friction coefficient μ_r, depending on material (steel) and wheels radii.

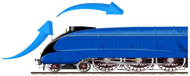
The problem

The train is characterized by:

1. the mass M;

2. the pulling force of the engine $U(t)$, represented by the function $u(t) = U(t)/M$;

3. the braking force $B(t)$, represented by the function $b(t) = B(t)/M$;



4. the aerodynamic parameters c_1 and c_2 corresponding to the air resistance, standing at terms proportional to velocity and its square respec- tively, already divided by M;
5. the maximum power developed by the engine, P_0, represented by $p_0 = P_0/M$.

The problem

The dynamics of the train is described by the following system of differential equations (g - gravity acceleration):

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = u(t) - b(t) - g(1 + \kappa_1 x_2^4)\mu_r - gs(x_1) - c_1 x_2 - c_2 x_2^2.$$

Our goal is to find the energy-optimal controls $u(t)$ and $b(t)$ transferring the train from standstill at $x_1 = 0$ to standstill at $x_1 = L$ in a given time T, namely to minimize the functional

$$V(u, b) = \int_0^T u(t) x_2(t) dt,$$

taking into account all additional conditions.

The problem

In connection with this problem there arise the following partial tasks.

1. Given the noisy measurements of the elevation $h(x)$ at the points along the track, represent the function $s(x)$ in a reasonable way.
2. Find the approximate controls.
3. Check, if the application of optimal controls is worthwhile, i.e. they really give smaller energy consumption in comparison to a reasonably good heuristic strategy.

1 Approximation of the slope

We assume, that we have the set of elevation measurements

$$(x_i, h(x_i)), \quad i = 1, \ldots, M, \quad x_i \in [0, L]$$

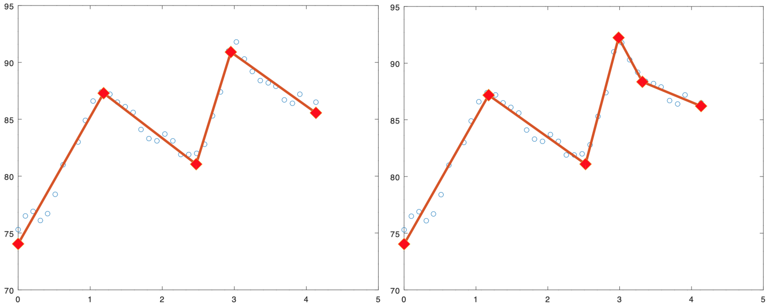
It seems, that the clever parametrization of the approximating function $h_{app}(x)$ could use (for a given N), the following set of parameters $z \in \mathbb{R}^{2N-1}$:

$$z = [h_0, s_1, \ldots, s_{N-1}, d_1, \ldots, d_{N-1}],$$

where h_0 - initial value at $x = 0$, s_i - slope on i-th segment, d_i - length of the i-th subinterval.
As a result, we have the following optimization problem:

$$\sum_{j=1}^{M} (h_{app}(z; x_j) - h(x_j))^2 \rightarrow \min$$

1 Approximation of the slope

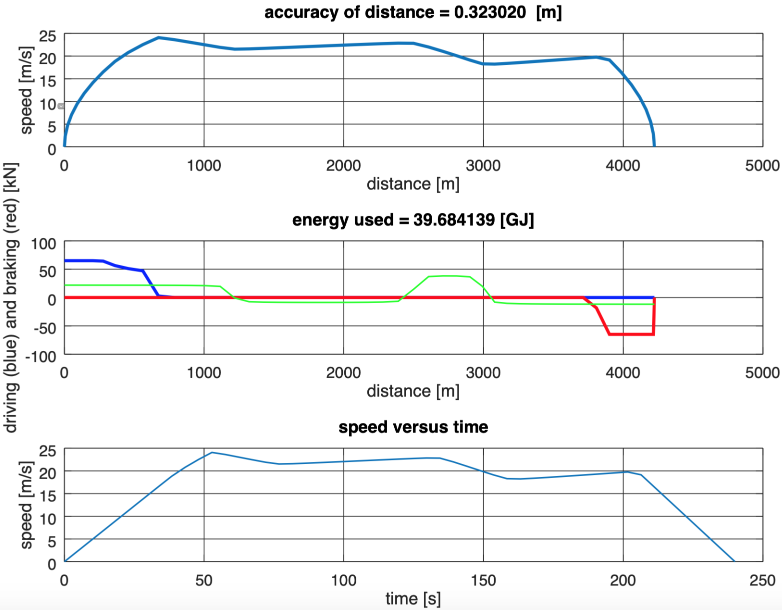


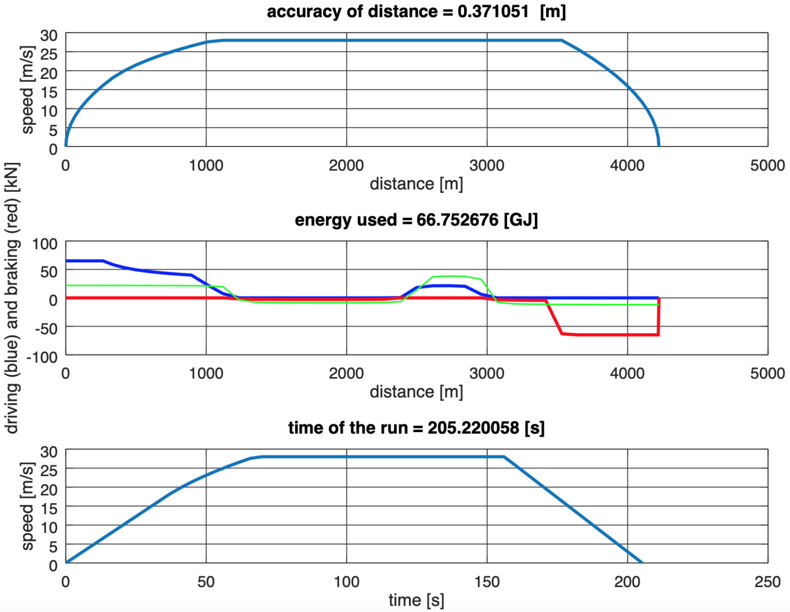
The piecewise linear approximation of a elevation for $N = 5$ (left) and $N = 6$ (right) and real measurements.

2 Approximate solution

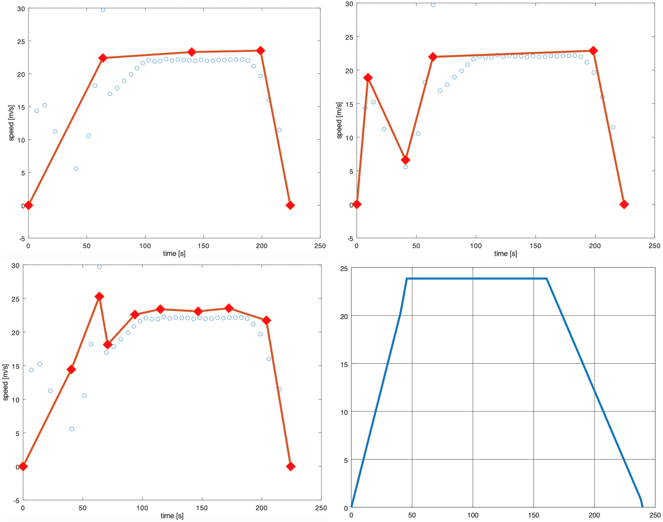
We have tested the reliability of the method on several examples of real tracks. In one of such cases we had:

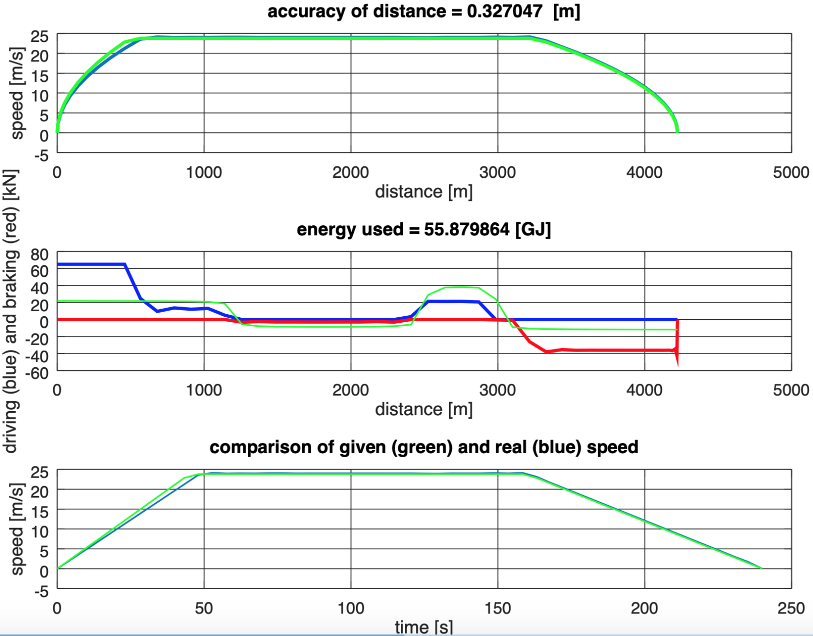
- the total length $L = 4223m$ divided by the slope approximation procedure into 4 subintervals $L_1 = 1200m$, $L_2 = 1300m$, $L_3 = 500m$, $L_4 = 1223m$;
- the approximate slopes represented by $s_1 = 0.00110$, $\delta_1 = s_2 - s_1 = -0.0156$, $\delta_2 = s_3 - s_2 = 0.0250$ and $\delta_3 = s_4 - s_3 = -0.0264$;
- train mass $M = 1.06e5[kg]$, rolling friction $\mu_r = 0.0015$, static friction $\mu_s = 0.0613$, $\kappa = c_2 = 0$, $c_1 = 0.1/M[1/s]$.
- the admissible speed $v_{ad} = 28[m/s]$, maximal power $P_0 = 1000[kW]$;
- the time of travel $T = 240[s]$.


accuracy of distance = 0.323020 [m]
speed [m/s]
distance [m]
energy used = 39.684139 [GJ]
driving (blue) and braking (red) [kN]
distance [m]
speed versus time
speed [m/s]
time [s]


accuracy of distance = 0.371051 [m]
speed [m/s]
distance [m]
energy used = 66.752676 [GJ]
driving (blue) and braking (red) [kN]
distance [m]
time of the run = 205.220058 [s]
speed [m/s]
time [s]

3 Velocity reproduction




accuracy of distance = 0.327047 [m]
speed [m/s]
distance [m]
energy used = 55.879864 [GJ]
driving (blue) and braking (red) [kN]
distance [m]
comparison of given (green) and real (blue) speed
speed [m/s]
time [s]

A word of caution

The approximation problem is strongly non-linear and non-convex. We should not expect global minimum and it is advisable to run procedure for each N several times with different (possibly random) starting points.

Thank you for your attention

References

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[2] Joel A E Andersson and Joris Gillis and Greg Horn and James B. Rawl- ings and Moritz Diehl: CasADi – A software framework for nonlinear optimization and optimal control, Mathematical Programming Compu- tation, In Press, 2018.

[3] A. Wachter and L. T. Biegler: On the Implementation of a Primal-Dual Interior Point Filter Line Search Algorithm for Large-Scale Nonlinear Programming, Mathematical Programming vol. 106, no. 1, pp. 25-57, 2006.