On problems and methods of coordinated scheduling and location

Jerzy Józefczyk, Mirosław Ławrynowicz, Grzegorz Filcek

Wrocław University of Science and Technology,
Division of Intelligent Decision Support Systems
Wrocław, Poland
1. Motivation
2. Scheduling sub-problem
3. Location sub-problem
4. Coordinated scheduling and location (ScheLoc)
5. Optimization schemes for ScheLoc
6. Selected results
7. Other cases and final remarks
Motivation (1/2)

Applications:
- Loading/unloading of ships at an embankment
- Planning of evacuation from danger zones
Motivation (2/2)
Scheduling sub-problem

Scheduling of jobs on parallel machines

\[ J = \{ J_1, J_2, \ldots, J_j, \ldots, J_n \} \] – set of \( n \) jobs

\[ M = \{ M_1, M_2, \ldots, M_i, \ldots, M_m \} \] – set of \( m \) machines

\( \sigma \in D_\sigma \) – schedule

\[
p = \begin{bmatrix} p_{ij} \end{bmatrix}_{i=1}^{m} \quad j=1, n \] – processing times

\[
w = \begin{bmatrix} w_j \end{bmatrix}_{j=1}^{n} \] – weights

\[
r = \begin{bmatrix} r_j \end{bmatrix}_{j=1}^{n} \] – release dates

\[
d = \begin{bmatrix} d_j \end{bmatrix}_{j=1}^{n} \] – due dates

\[ A_S = (p, d, w, r) \] – sequence of parameters

\[ [C_j(\sigma; A_S)]_{j=1}^{n} \] – completion times of jobs

\[ \bar{Q}_s([C_j(\sigma; A_S)]_{j=1}^{n}) \sqcup Q_s(\sigma; A_S) \] – job scheduling criteria

\[ \min_{\sigma \in D_\sigma} Q_s(\sigma; A_S) \] – Sub-problem PS
Location sub-problem (1/2)

Location of machines

\[ a = \begin{bmatrix} a_j \end{bmatrix}_{j=1,n} \] \quad \text{given locations of jobs}

\[ x = \begin{bmatrix} x_i \end{bmatrix}_{i=1,m} \] \quad \text{sought locations of machines}

\[ v = \begin{bmatrix} v_j \end{bmatrix}_{j=1,n} \] \quad \text{speeds of job movements}

\[ c = \begin{bmatrix} c_{ij} \end{bmatrix}_{i=1,m}^j_{j=1,n} \] \quad \text{unit movement/transportation costs}

\[ g = \begin{bmatrix} g_i \end{bmatrix}_{i=1,m} \] \quad \text{unit location/investment costs}

\[ A_L = (a, v, c, g) \] \quad \text{sequence of parameters}

Sub-problem PL \quad \min_{x \in D_x} Q_{LL}(x; A_L) \quad (mainly, location and/or movement costs)

where \( D_x \) represents available machines' positions taken from the discrete or continuous area
Location sub-problem (2/2)

\[ r_{ij}(x_i) = \rho_j + v_j^{-1} d(a_j, x_i) \] – generalized release dates, where \( d(\cdot) \) – distance

\[ D_{\sigma} \to D_{\sigma x} \quad Q_S(\sigma; A_s) \to Q_{SL}(\sigma, x; A_s) \]

\( \alpha | \beta | \gamma | \delta \) – extended Graham's notation

\( \beta: r_j \text{ V, } \quad r_j^- \leq r_j \leq r_j^+ \)

\( \delta: \) – UFLP  \hspace{1cm} (full UFLP)

\hspace{1cm} – UFLP_T  \hspace{1cm} (UFLP with the restriction to a transportation cost)

\hspace{1cm} – UFLP_I  \hspace{1cm} (UFLP with the limitation to an investment cost)

\hspace{1cm} – P_MEDIAN  \hspace{1cm} (p-median problem)

\hspace{1cm} – CFLP  \hspace{1cm} (facility location with continuous location area)
Table 1. The track record on coordinated scheduling-location

<table>
<thead>
<tr>
<th>Problem</th>
<th>Description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( r_j ) ( V )</td>
<td>( C_{\text{max}} )</td>
</tr>
<tr>
<td>1</td>
<td>( r_j ) ( V )</td>
<td>( C_{\text{max}} )</td>
</tr>
<tr>
<td>1</td>
<td>( p_j = p, r_j ) ( V )</td>
<td>( C_{\text{max}} )</td>
</tr>
<tr>
<td>( P )</td>
<td>( r_j ) ( V )</td>
<td>( C_{\text{max}} )</td>
</tr>
<tr>
<td>( P )</td>
<td>( r_j ) ( V )</td>
<td>( C_{\text{max}} )</td>
</tr>
<tr>
<td>( P )</td>
<td>( r_j ) ( V )</td>
<td>( \sum C_j )</td>
</tr>
<tr>
<td>( R )</td>
<td>( r_j ) ( V )</td>
<td>( C_{\text{max}} )</td>
</tr>
<tr>
<td>( R )</td>
<td>( r_j ) ( V )</td>
<td>( C_{\text{max}} )</td>
</tr>
<tr>
<td>( P )</td>
<td>( r_j ) ( V )</td>
<td>( \sum C_j )</td>
</tr>
</tbody>
</table>
Coordinated scheduling and location (ScheLoc) (2/2)

Decision-making problem

*Given:*
- model of a decision-making plant
- requirement for a result of a decision (evaluation of a decision)

*Determine:*
- decision fulfilling a requirement
Fig. 2 Sequential approach for solving ScheLoc /S_Seq/
Sequential optimization scheme S_Seq

**Require:** J, M, A_S, A_L, D_{\sigma x}, D_x

**Ensure:** \( \bar{x}, \bar{\sigma}, \bar{Q}_L \subseteq Q_L(\bar{x}; A_L), \bar{Q}_{SL} \subseteq Q_{SL}(\bar{\sigma}, \bar{x}; A_S) \),

1. Solve the locational problem \( \min_{x \in D_x} Q_L(x; A_L) \) to obtain \( \bar{x} \) and \( \bar{Q}_L \).
2. Calculate \( r_{ij} = \rho_j + v_j^{-1}d(a_j, \bar{x}_i), i = 1, m, j = 1, n \).
3. Set the release dates of jobs using the selected operator \( \Phi \) to have \( r_j = \Phi(r_{i1}, ..., r_{mj}) \).
4. Solve the job scheduling problem with different release dates \( r_j \min_{\sigma \in D_{\sigma x}} Q_{SL}(\sigma, \bar{x}; A_S) \) to determine \( \bar{\sigma} \) and \( \bar{Q}_{SL} \).
Optimization schemes for ScheLoc: /S_Jnt/ (4/9)

Fig. 3 Joint approach for solving ScheLoc with an only criterion $Q_{SL}$ /S_Jnt/
Joint optimization scheme S_Jnt

**Require:** $J, M, A_S, A_L, D$

**Ensure:** $\hat{\sigma}, \hat{x}, \hat{Q}_{SL} \subseteq Q_{SL}(\sigma, x; A_S)$

1. Solve the joint scheduling-location problem $\min_{(\sigma, x) \in D} Q_{SL}(\sigma, x; A_S)$ to obtain $\hat{\sigma}, \hat{x},$ and $\hat{Q}_{SL}.$
Fig. 4 Bi-criteria joint approach for solving ScheLoc.
Optimization schemes for ScheLoc: /S_Jnt_B/  

Joint bi-criteria optimization scheme S_Jnt_B

**Require:**  \( J, M, A_S, A_L, D \)

**Ensure:** Set of \( \lambda \) pairs of decisions \( \text{Arg}_{PF} = \{(\sigma_l^*, x_l^*): l = 1, \lambda\} \) and corresponding Pareto front \( PF = (Q_l^* \sqcup [Q_{SL,l}^*, Q_{L,l}^*])^T: Q_{SL,l}^* < Q_{SL,l-1}^*, l = 2, \lambda \). 

1. Solve the joint scheduling-location problem \( \min_{(\sigma, x) \in D} \left[ Q_{SL}(\sigma, x; A_S) \right] \) to have \( \text{Arg}_{PF} \) and \( PF \).
Fig. 5 Uncertain approach for solving ScheLoc – addressing as an uncertain version of PS /S_Unc/
Uncertain optimization scheme $S_{Unc}$

**Require:** $J$, $M$, $A_S$, $D_\sigma$, $R$

**Ensure:** $\sigma''$, $r^\sigma$, $z(\sigma'')$, $Q_S(\sigma'', r^\sigma)$ \(\subseteq Q''_S\)

1. Solve \(\min_{\pi \in D_S} \hat{Q}_S(\pi, r)\) to have \(Q'_S(r)\).

2. Obtain the worst-case scenario as \(r^\sigma = \arg \max_{r \in R} [\hat{Q}_S(\sigma, r) - Q'_S(r)]\) and calculate \(z(\sigma) = \hat{Q}_S(\sigma, r^\sigma) - Q'_S(r^\sigma)\).

3. Solve \(\min_{\sigma \in D_\sigma} z(\sigma)\) to obtain $\sigma''$, $z(\sigma'')$, and $Q''_S$. 
Selected results: Case $P | r_j \in V | \sum C_j | UFLP_I$ (1/4)


Discrete locations, the Euclidean distances

$\mu$ locations ready for deploying $m$ machines

$$A_L = (a, v, g)$$

$$Q_{SL} (\sigma, x; A_S) \rightarrow \quad Q_{SL} (s, x; A_S) = \sum_{i=1}^{\mu} \sum_{k=1}^{n} x_i C_{ik} (s, x)$$

s.t.

$$1 \leq \sum_{i=1}^{\mu} x_i \leq \mu,$$

$$\sum_{i=1}^{\mu} \sum_{k=1}^{n} s_{ijk} = 1, \quad j = 1, 2, ..., n,$$

$$\sum_{j=1}^{n} s_{ijk} \leq x_i, \quad i = 1, 2, ..., \mu, \quad k = 1, 2, ..., n,$$

$$C_{ik} (s, x) \geq \sum_{j=1}^{n} (r_j + p_j) s_{ijk}, \quad i = 1, 2, ..., \mu, \quad k = 1, 2, ..., n,$$

$$C_{ik} (s, x) \geq C_{i,k-1} (s, x) + \sum_{j=1}^{n} p_j s_{ijk}, \quad i = 1, 2, ..., \mu, \quad k = 2, 3, ..., n.$$
Selected results: Case P | r_j _V | \sum C_j | UFLP_I

(2/4)

Algorithm ALG_BC

Require: Data of BC_ScheLoc: J , B , A , \mu , p_j , \rho_j , v_j , c_i , and parameters of the algorithm: \varphi_m , \varphi_r , \alpha_i , \alpha \tilde{a} , \alpha \Gamma , \Gamma_{max}.

Ensure: Set of non-dominated solutions S_{pf} and the Pareto front PF.

1: \text{i} := 1, \text{z} := 1, \tilde{z} := 1, S_m(iz) := \emptyset

2: \text{while } i \leq \mu \text{ do}

3: \text{Generate the initial population } S_i \text{ of } \alpha_i \text{ solutions for exactly } i \text{ executors and set } \bar{S}_i(0) := \emptyset

4: \text{while } \text{iz} < \Gamma_{max} \text{ or } \tilde{z} < \Gamma \text{ do}

5: \text{Perform a non-dominated sorting and crowding-distance assignment.}

6: \text{Use a crowded comparison operator for a selection.}

7: \text{Generate the set } \bar{S}_i(z) \text{ of offsprings using the crossover (CPC-2/OX2) and mutation (SIM/SM) operators.}

8: \text{for each } l \text{th element in } \bar{S}_i(z) \text{ that is non-dominated by any element in } S_m(iz) \text{ do}

9: \text{ } S_m(iz) := S_m(iz) \cup \bar{E}_l(z)

10: \text{end for}

11: \text{if } |\bar{S}_i(z)| = |\bar{S}_i(z - 1)| \text{ then } \tilde{z} := \tilde{z} + 1 \text{ else } \tilde{z} := 1 \text{ end if}

13: \text{if } \Gamma_{max} \text{ mod } z = 0 \text{ or } \tilde{z} := \Gamma \text{ then set } i := i + 1, z := 1 \text{ and go to 2, end if}

16: \text{end while}

17: \text{Set } z := z := z + 1.

18: \text{end while}

19: \text{return } S_{pf} = \left\{ (x_{PF,l} = f_x(E_{l,l}), y_{PF,l} = f_y(E_{l,l})) \right\}, l = 1, 2, ..., L(iz)

\text{and } \text{PF} = \left\{ (q_i^{(0)}(x_{PF,l}, y_{PF,l}), q_i^{(2)}(y_{PF,l})) \mid \left[ q_i^{(0)}, q_i^{(2)} \right] : q_i^{(0)} < q_i^{(2)}, l = 1, 2, ..., L(iz) \right\}
Table 2 Dependence of $Q_{SL,d}$, $Q_{L,d}$ and $Dist$ on $n$ and $\mu$ for both algorithms

<table>
<thead>
<tr>
<th>$(n, \mu)$</th>
<th>$Q_{SL,l}^*$</th>
<th>$Q_{L,l}^*$</th>
<th>$Q_{NS}^{SL,l}$</th>
<th>$Q_{NS}^{L,l}$</th>
<th>$Dist^*$</th>
<th>$Dist_{NS}^*$</th>
<th>$Dist_{NS}^* / Dist^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10, 4)</td>
<td>266.776</td>
<td>182</td>
<td>266.776</td>
<td>182</td>
<td>322.945</td>
<td>322.945</td>
<td>1.000</td>
</tr>
<tr>
<td>(15, 4)</td>
<td>432.052</td>
<td>285</td>
<td>432.052</td>
<td>285</td>
<td>517.584</td>
<td>517.584</td>
<td>1.000</td>
</tr>
<tr>
<td>(20, 4)</td>
<td>596.663</td>
<td>385</td>
<td>624.574</td>
<td>385</td>
<td>710.093</td>
<td>733.701</td>
<td>1.033</td>
</tr>
<tr>
<td>(25, 4)</td>
<td>866.800</td>
<td>385</td>
<td>956.164</td>
<td>385</td>
<td>948.455</td>
<td>1030.765</td>
<td>1.087</td>
</tr>
<tr>
<td>(10, 6)</td>
<td>265.559</td>
<td>177</td>
<td>265.559</td>
<td>177</td>
<td>319.140</td>
<td>319.140</td>
<td>1.000</td>
</tr>
<tr>
<td>(15, 6)</td>
<td>406.760</td>
<td>257</td>
<td>448.850</td>
<td>257</td>
<td>481.147</td>
<td>517.218</td>
<td>1.075</td>
</tr>
<tr>
<td>(20, 6)</td>
<td>458.741</td>
<td>406</td>
<td>477.497</td>
<td>452</td>
<td>612.600</td>
<td>725.684</td>
<td>1.185</td>
</tr>
<tr>
<td>(25, 6)</td>
<td>611.681</td>
<td>452</td>
<td>716.951</td>
<td>536</td>
<td>760.564</td>
<td>911.585</td>
<td>1.199</td>
</tr>
</tbody>
</table>

$Dist$ – distance between (0,0) and Pareto front /$Q_{SL,l}, Q_{L,l}$/$

NS – NSGS II-based algorithm
Selected results: Case $P | r_{j-V} | \sum C_j | \text{UFLP}_I$ (4/4)
Selected results: Cases $R \mid r_j \_ V \mid C_{\text{max}} \mid P_{\text{MEDIAN}}$ and $R \mid r_j \_ V \mid C_{\text{max}}$ (1/5)


$R \mid r_j \_ V \mid C_{\text{max}} \mid P_{\text{MEDIAN}} \rightarrow \text{Scheme S\_Seq}$

discrete locations, the Euclidean distances

$\mu$ locations ready for deploying $m$ machines

$A_L = (a, v)$

**Sub-problem PL**  (*p*-median problem)

$Q_L(x; A_L) = \sum_{i=1}^{\mu} \min_{x \in D_x} r_{ij} \ r_{ij}(x_i) = \rho_j + v_j^{-1}d(a_j, x_i)$

Selected results: Cases $R | r_j - V | C_{\text{max}} | \text{P-MEDIAN}$ and $R | r_j - V | C_{\text{max}}$ (2/5)

Sub-problem PS

$A_S = (p, r)$

$Q_{SL}(\sigma, x; A_S) = \max_{1 \leq i \leq m} C_{\sigma_i}^n(x_i)$

$\sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_i, \ldots, \sigma_m\}$

$\sigma_i = (\sigma_i^1, \sigma_i^2, \ldots, \sigma_i^1, \ldots, \sigma_i^n)$

$D_{\sigma x} = \{\sigma: \sigma_i^1 \neq \sigma_i^2, \sum_{i=1}^m n_i = n\}$

PSO–based solution algorithm

Selected results: Cases \( R | r_j V | C_{\text{max}} | P_{\text{MEDIAN}} \) and \( R | r_j V | C_{\text{max}} \)

\[
\begin{align*}
R | r_j V | C_{\text{max}} & \rightarrow \text{Scheme S\_Int} \\
\end{align*}
\]

Joint algorithm of job scheduling and location

Location of executors and job scheduling plant \( r(x) \)

Algorithm TSMA. Tabu Search based Memetic algorithm

 Require: Data: \( D = \{ M, Y, \bar{Y}, J, S, V, \rho \} \) and parameters of the algorithm: \( \alpha_{\text{init}}, \alpha_{\text{par}}, \varphi_m, \varphi_r, \Gamma, \Gamma_{\text{max}}, \rho, \bar{\rho}, \kappa_{\text{bin}}, \kappa_{\text{integer}} \)

 Ensure: Heuristic solution \( X \) and \( \Pi \) as well as \( C_{\text{max}}^{TSMA} \).

1. Generate the initial population \( \tilde{E}(z) \) for \( z := 1 \) and set \( \mathbf{E}^* := \mathbf{E}_{\text{best}}(z) \).
2. \textbf{while} stop condition defined by \( \left( \Gamma, \Gamma_{\text{max}}, \mathbf{E}^* \right) \) is not satisfied
3. Evaluate \( \tilde{o}(z) \) by population \( \tilde{E}(z) \).
4. Select parents \( \tilde{E}_{\text{sus}}(z) \) with the use of Stochastic Universal Sampling.
5. Generate the set \( \tilde{E}_{\text{off}}(z) \) of offsprings using the recombination (CPC-2/OX1) and mutation operators.
6. Run the Local Search Heuristic with arguments: \( \tilde{o}^{\text{off}}(z), D \) and return \( \mathbf{E}^{TS} \).
7. \textbf{if} \( C_{\text{max}}(\mathbf{E}^*) > C_{\text{max}}(\mathbf{E}^{TS_{\text{best}}}) \) then \( \mathbf{E}^* := \mathbf{E}^{TS_{\text{best}}} \) \textbf{end if}
8. Set \( z := z + 1 \) and \( \tilde{E}(z) := \tilde{E}^{TS} \cup \tilde{E}_{\text{off}_{\text{best}}}(z) \).
9. \textbf{end while (2)}
10. \( X := f_X(\mathbf{E}^*; Y), \quad \Pi := f_\Pi(\mathbf{E}^*), \quad C_{\text{max}}^{TSMA} := C_{\text{max}}(\Pi, X) \)
Table 3 Dependence of $Q_{SL}$ and $t$ on $n$ for joint and sequential approaches

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\hat{Q}_{SL}$</th>
<th>$\tilde{Q}_{SL}$</th>
<th>$\delta_Q$</th>
<th>$\hat{t}_{SL}$ [s]</th>
<th>$\tilde{t}_{SL}$ [s]</th>
<th>$\delta_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>39.764</td>
<td>52.780</td>
<td>32.733</td>
<td>9.221</td>
<td>7.035</td>
<td>-23.707</td>
</tr>
<tr>
<td>200</td>
<td>71.551</td>
<td>100.954</td>
<td>41.094</td>
<td>16.934</td>
<td>14.697</td>
<td>-13.210</td>
</tr>
<tr>
<td>300</td>
<td>146.625</td>
<td>188.781</td>
<td>28.751</td>
<td>25.755</td>
<td>26.968</td>
<td>4.710</td>
</tr>
<tr>
<td>400</td>
<td>235.912</td>
<td>297.873</td>
<td>26.264</td>
<td>34.777</td>
<td>36.089</td>
<td>3.773</td>
</tr>
<tr>
<td>500</td>
<td>300.651</td>
<td>369.762</td>
<td>22.987</td>
<td>41.048</td>
<td>47.114</td>
<td>14.778</td>
</tr>
<tr>
<td>600</td>
<td>348.684</td>
<td>462.543</td>
<td>32.654</td>
<td>49.912</td>
<td>58.281</td>
<td>16.768</td>
</tr>
<tr>
<td>700</td>
<td>405.646</td>
<td>527.986</td>
<td>30.159</td>
<td>58.995</td>
<td>72.092</td>
<td>22.200</td>
</tr>
<tr>
<td>800</td>
<td>489.871</td>
<td>659.451</td>
<td>34.617</td>
<td>67.665</td>
<td>83.668</td>
<td>23.650</td>
</tr>
<tr>
<td>900</td>
<td>579.543</td>
<td>778.992</td>
<td>34.415</td>
<td>80.019</td>
<td>99.769</td>
<td>24.682</td>
</tr>
<tr>
<td>1000</td>
<td>645.671</td>
<td>868.621</td>
<td>34.530</td>
<td>88.894</td>
<td>117.249</td>
<td>31.898</td>
</tr>
</tbody>
</table>

\[
\delta_Q = \frac{\tilde{Q}_{SL} - \hat{Q}_{SL}}{\hat{Q}_{SL}} \times 100\%
\]

\[
\delta_t = \frac{\tilde{t}_{SL} - \hat{t}_{SL}}{\hat{t}_{SL}} \times 100\%
\]
Property. The optimization scheme $S_{\text{Seq}}$ does not outperform the optimization scheme $S_{\text{Int}}$, i.e.,

$$ Q_{SL}(\tilde{\sigma}, \tilde{x}; A_S) \geq Q_{SL}(\tilde{\sigma}, \hat{x}; A_S) $$

Justification:

$$ Q_{SL}(\tilde{\sigma}, \hat{x}; A_S) = \min_{(\sigma, x) \in D} Q_{SL}(\sigma, x; A_S) \leq \min_{(\sigma, x) \in D'} Q_{SL}(\sigma, x; A_S) = \min_{\sigma \in \sigma_{\tilde{x}}} Q_{SL}(\sigma, \tilde{x}; A_S) = Q_{SL}(\tilde{\sigma}, \tilde{x}; A_S), $$

where $\tilde{x} = \arg\min_{x \in D_x} Q_L(x; A_L)$, $D' = \{(\sigma, x) : \sigma \in D_{\sigma_{\tilde{x}}} \wedge \tilde{x} = \arg\min_{x \in D_x} Q_L(x; A_L)\}$, and $D' \subseteq D$. 

Release dates belong to given intervals, i.e., \( r_j \in [r_{j^-}, r_{j^+}] \).

How to determine intervals?

\[
\hat{Q}_S(\sigma, r) \subseteq C_n(\sigma, r)
\]

where \( C_k(\sigma, r) = p_{\sigma(k)} + \max\left\{ C_{k-1}(\sigma, r), r_{\sigma(k)} \right\}, \quad k = 2, 3, \ldots, n, \quad C_0(\sigma, r) = 0
\]

\[
\hat{Q}_S(\sigma, r) - Q'_S(\sigma'_r) \quad \text{regret}
\]

where \( \sigma' = \arg \min_{\pi \in D_\pi} Q(\pi, r) \) – optimal solution for fixed scenario \( r \)

\( D_\pi \) – set of permutations without repetitions

\[
Q'_S = \min_{\pi \in D_\pi} Q(\pi, r)
\]

\[
z(\sigma) = \max_{r \in R} [\hat{Q}_S(\sigma, r) - Q'_S(\sigma'_r)] \quad \text{maximum regret for given } \sigma
\]

\[
r^\sigma = \arg \max_{r \in R} [\hat{Q}_S(\sigma, r) - Q'_S(r)] \quad \text{worst-case scenario}
\]

\[
\min_{\sigma \in D_\sigma} z(\sigma) \quad \sigma'' = \arg \min_{\sigma \in D_\sigma} z(\sigma) \quad \text{solution}
\]
Table 4 Dependence of makespan and computational time on $n$ for S_Jnt and S_Unc, $\mu = 50$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\hat{Q}_{SL}$</th>
<th>$Q_s(\sigma'')$ [ms]</th>
<th>$t_s$ [ms]</th>
<th>$\hat{Q}_{SL}$</th>
<th>$Q_s(\sigma'')$ [ms]</th>
<th>$t_s$ [ms]</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>2741.530</td>
<td>2776.632</td>
<td>8</td>
<td>39</td>
<td>556.55</td>
<td>11</td>
</tr>
<tr>
<td>80</td>
<td>5719.114</td>
<td>5747.075</td>
<td>10</td>
<td>296</td>
<td>709.33</td>
<td>16</td>
</tr>
<tr>
<td>120</td>
<td>8737.921</td>
<td>8785.887</td>
<td>26</td>
<td>1241</td>
<td>811.91</td>
<td>29</td>
</tr>
<tr>
<td>160</td>
<td>11505.807</td>
<td>11533.889</td>
<td>29</td>
<td>4410</td>
<td>987.38</td>
<td>39</td>
</tr>
<tr>
<td>200</td>
<td>14482.500</td>
<td>14503.128</td>
<td>31</td>
<td>6712</td>
<td>1108.99</td>
<td>55</td>
</tr>
<tr>
<td>240</td>
<td>17534.556</td>
<td>17557.584</td>
<td>47</td>
<td>12744</td>
<td>1293.77</td>
<td>74</td>
</tr>
<tr>
<td>280</td>
<td>20638.523</td>
<td>20680.748</td>
<td>63</td>
<td>22208</td>
<td>1671.41</td>
<td>92</td>
</tr>
<tr>
<td>320</td>
<td>23102.561</td>
<td>23138.121</td>
<td>78</td>
<td>35916</td>
<td>1822.76</td>
<td>111</td>
</tr>
<tr>
<td>360</td>
<td><strong>26457.697</strong></td>
<td><strong>26457.697</strong></td>
<td>92</td>
<td>49812</td>
<td><strong>2098.41</strong></td>
<td>133</td>
</tr>
<tr>
<td>400</td>
<td><strong>29299.100</strong></td>
<td><strong>29299.100</strong></td>
<td>110</td>
<td>71251</td>
<td><strong>2223.91</strong></td>
<td>161</td>
</tr>
</tbody>
</table>

Selected results: Uncertain case $1|_{r_{j}^{-}} \leq r_{j} \leq r_{j}^{+}| C_{max}$ (2/2)
Connection of the deployment of machines with the assignment of jobs to machines.

Next, $m$ separate job scheduling sub-problems on single machines.

Advantage – substantial simplification of PS.

The detailed analysis is an open issue, in particular, on the connection between location and assignment.


Clusters of jobs are determined around launched sites for machines.
Machine-dependent release dates $r_{ij}$ are figured in S_Seq.

PS needs a single release date $r_j$.

The substantiation other than max, min or mean value needs research.

We would have a new issue with machine-dependent release dates; the comparison to similar known problems required.
How to calculate $r_{ij}$?

Applied proposition: $r_{ij}(x_i) = \rho_j + v_j^{-1}d(a_j, x_i)$

Other methods for $S_{Unc}$, e.g., the probabilistic one and release date values acquisition
Thank you for your attention.