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Evaluation of digital services by voting towards MCDM: the pedagogy of the Saari triangle

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SALUTING THE TWO PROFESSORS (EMERITI)

[DONALD G. SAARI](https://www.math.uci.edu/~dsaari/)
https://www.math.uci.edu/~dsaari/

[HANNU NURMI](https://www.utu.fi/en/people/hannu-nurmi)
https://www.utu.fi/en/people/hannu-nurmi

I)Some background INFO on voting and decision-making

- Voting procedures describe the manner in which the preferences of individuals are combined to produce a collective decision.
- ASSUMPTION:
  - Each voter has a linear order on the set of candidates from the most desirable candidate to the least desirable one.
  - Each voter votes according to her the true preferences (i.e. sincerely).
- A voting procedure is defined by **1) the type of a vote** , and **2) the aggregation rule.**
- Voting methods are numerous, and using different methods the same group of voters can end up with different outcomes.
- The more candidates and voters, the more complexities and discrepancies arise **("The curse of dimensionality").**

II)Some background INFO on voting and decision-making

Especially **positional procedures** are complex: voting outcomes can change when the amount of candidates (alternatives) are either added or dropped.

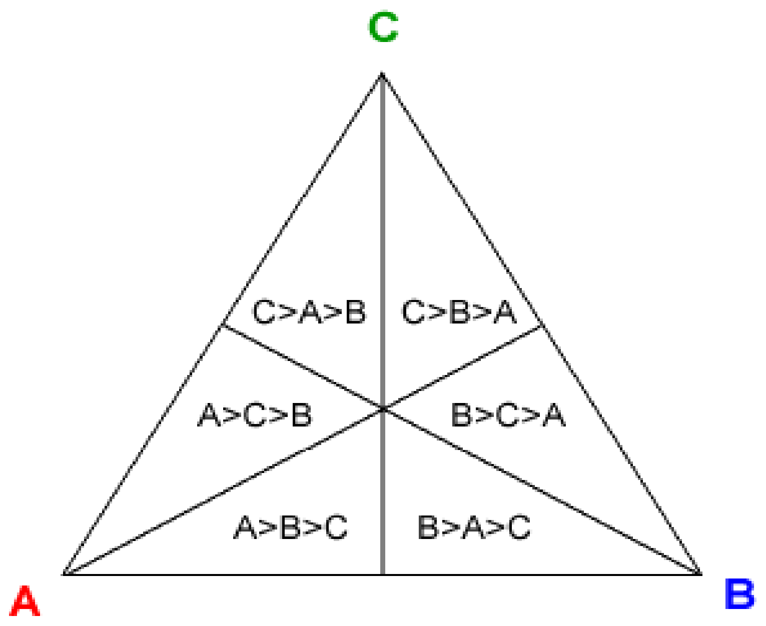
Varying the choice of positional methods outcomes become most contradictory although the marked ballots fix the voters' (decision-makers') opinions: with some methods some alternatives win while with others they may be bottom-ranked.

II)Some background INFO on voting and decision-making

An election or decision outcome not necessarily reveals the true preferences of the voters but moreover the choice of an election rule. Problems arise when a voting rule ignores crucial but available information about the profile.

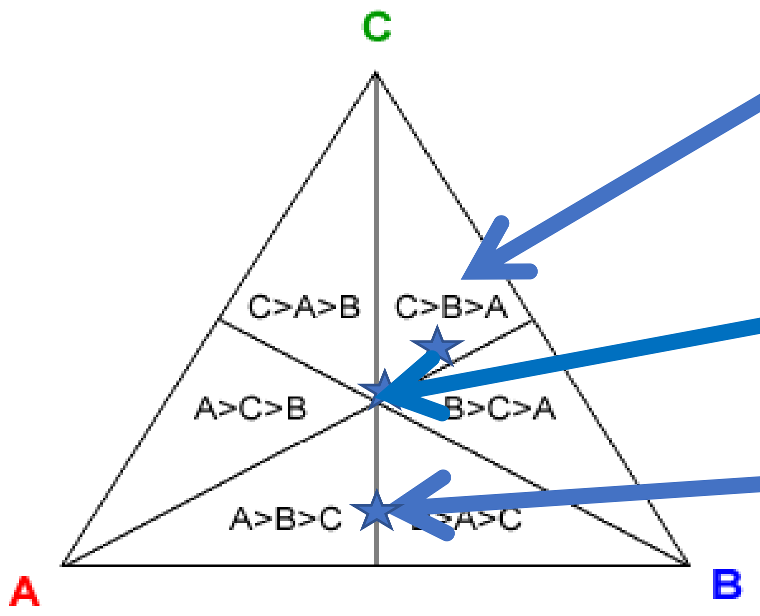
As voting methods are prototypes of general aggregation rules, same kind of inconsistencies may occur in other disciplines [**multi-criteria decision-making (MCDM), economics, statistics** …] as well.

The Saari triangle



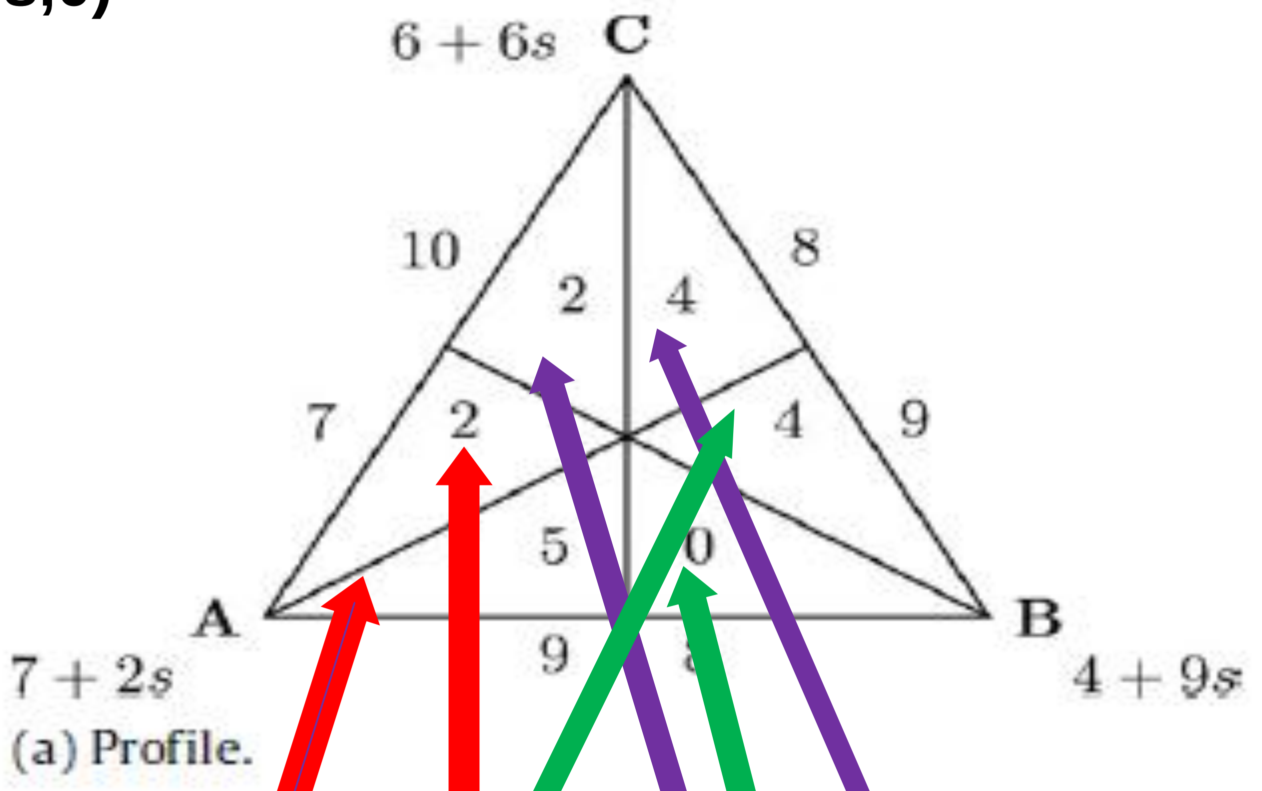
- With a choice set of three alternatives **A**, **B**, and **C**, there are six possible (strict) preference profiles for each mode:
- 1. A > B > C; 2. A > C > B; 3. C > A > B; 4. C > B > A; 5. B > A > C; 6. B > C > A
- These profiles can be represented geometrically in a triangle, with each vertex representing a choice option.
- **The triangle can then be divided into six equally large (ranking) regions, which represent the specific profiles. [with the ties counted in – a total of thirteen (13) regions is defined**

The Saari triangle



- The ordinal ranking of a point in the triangle comes from its distances to the vertices where **"the closer the better"**
- The midpoint of the triangle represents **a complete tie** between the alternatives with equal share of votes for each. The median line initiating from any of the vertices dividing the opposite side of the triangle to parts of equal length, represents **a tie between the two other alternatives.**
- **A positional election with the three candidates A, B, and C is defined by the (normalized) voting vector (s)=w(1),w(2),w(3)=(1,s,0), where s , 0≤s≤1, is a specified weight for a second-ranked alternative (i.e. candidate). For a given voting procedure each choice option receives a number of points reflecting its ranking.**

One realization of the Saari triangle :
the voting vector (s)=w(1),w(2),w(3)=(1,s,0)



1)WITH THE PLURALITY RULE
("VOTE FOR ONE")"
S=0, the positional rule reduces to the **plurality** method **W(PL)=(1,0,0)**
A WINS

2)WITH THE ANTIPLURALITY RULE
("VOTE FOR TWO")
S=1,the **antiplurality** method gives the results **W(APL)=(1,1,0),**
i.e. against the 3rd-place alternative
B WINS

3)WITH THE BORDA COUNT (2,1,0)
S=½ gives the **Borda count W(BC)=(2,1,0),**
i.e. 2 points for each 1st-place vote, 1 point for each 2nd-place vote - 0 points for 3rd-place votes. **C WINS**

AN EXAMPLE OF A PREFERENCE PROFILE

NUMBER OF VOTES	RANKING	NUMBER OF VOTES	RANKING
5	A>B>C	4	C>B>A
2	A>C>B	2	C>A>B
4	B>C>A		

0=ZERO VOTES; B>A>C

I)The procedure line: the methodological essence

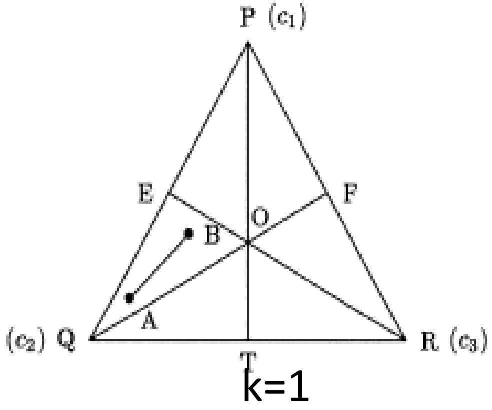
The plurality and antiplurality outcomes define the endpoints of the procedure line, and the w(s) outcome is the point on the procedure line which is 2(s) of the distance from the plurality endpoint. Experimenting with the placement of lines

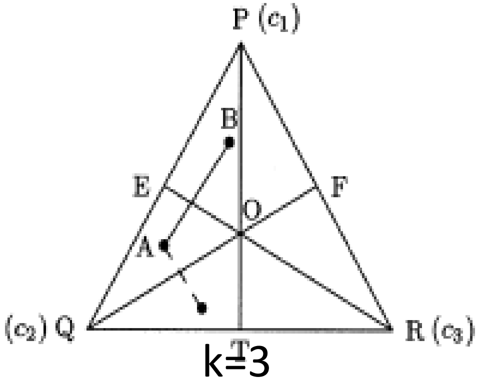
the value of k

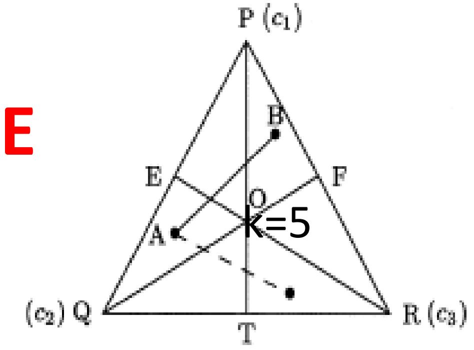
is restricted between unity and seven .

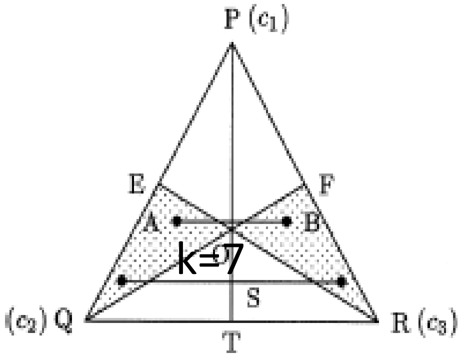
SIC!

A PROCEDURE LINE CAN CROSS NO MORE THAN SEVEN (7) OF THE THIRTEEN (=13) RANKING REGIONS.









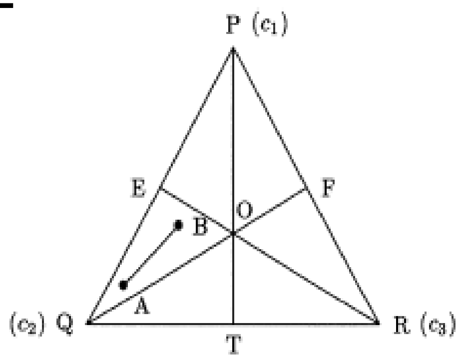
II)The procedure line: the methodological essence

If a procedure line has its endpoints in regions with reversed strict rankings, then the line either passes through the complete indifference point (so, k = 3) or through k =7 regions.

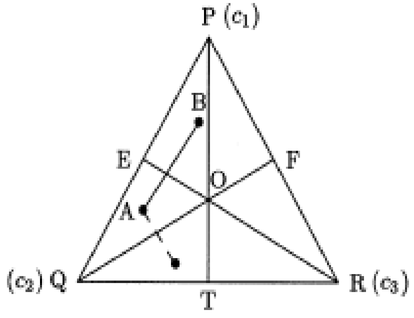
If k has an even value (k=2, 4 or 6), then the geometry requires an endpoint to be on an indifference line of the representation triangle (i.e. a pairwise tie).

ONLY THE ENDPOINTS MATTER!

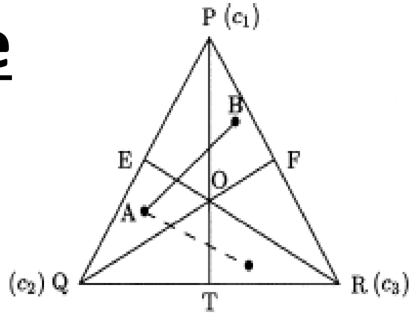
THE PLURALITY AND ANTIPLURALITY POINTS.



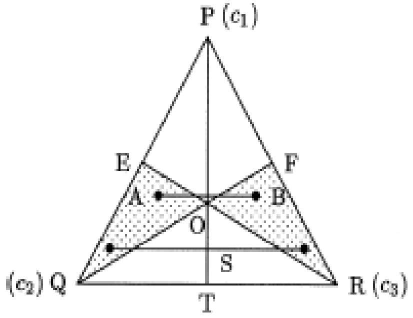
k=1



k=3



k=5



k=7

III)The procedure line: the methodological essence

IN CONCLUSION

THE PROBALITY TO GET 7 DIFFERENT POSITIONALS OUTCOMES IS= 0.06.

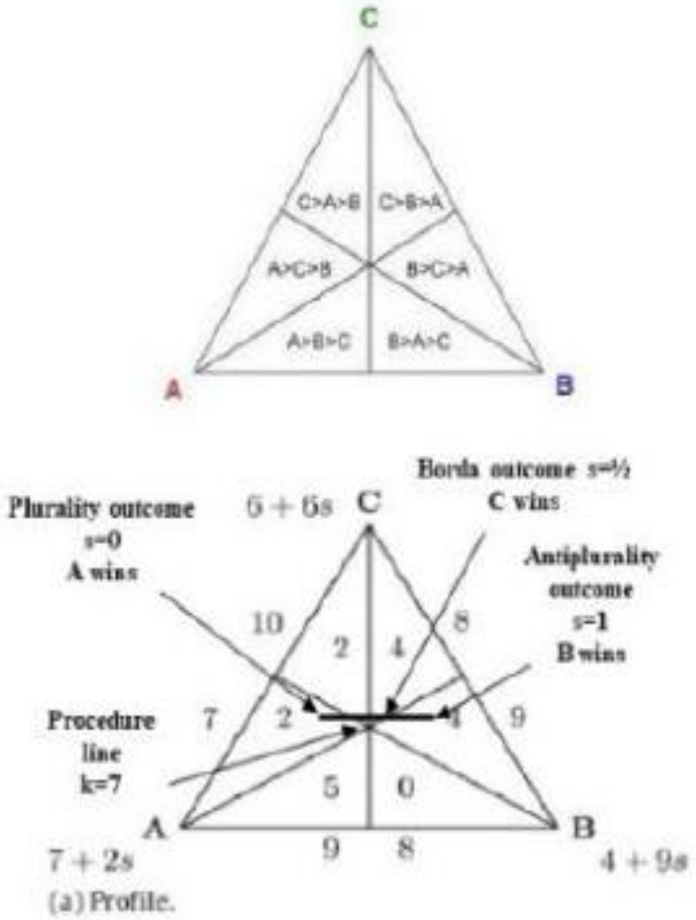
THE PROBALITY TO GET 5 DIFFERENT POSITIONALS OUTCOMES IS= 0.19.

THE PROBALITY TO GET 3 DIFFERENT POSITIONALS OUTCOMES IS= 0.44.

THE PROBALITY TO GET THE SAME (K=1) OUTCOMES IS= 0.31.

THEY ALL ADD UP TO ONE (0.06+0.19+0.44+0.31 = 1).

AN EXAMPLE OF A PREFERENCE PROFILE [15]			
NUMBER OF VOTES	RANKING	NUMBER OF VOTES	RANKING
5	A ≻ B ≻ C	4	C ≻ B ≻ A
2	A ≻ C ≻ B	2	C ≻ A ≻ B
4	B ≻ C ≻ A	0	B ≻ A ≻ C



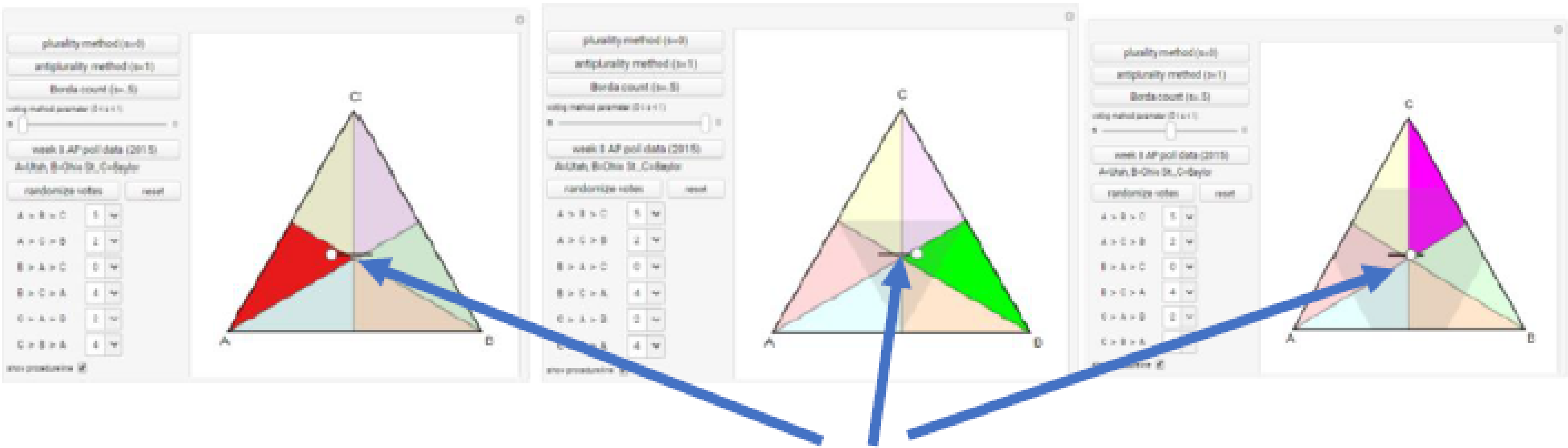
PAIRWISE RANKINGS [15] ⟶ CYCLIC PREFERENCES ⟶ NO RATIONAL SOLUTION!
A ≻ B ≡ 9 ≻ 8 ≡ (2+2+5=9) ≻ (4+4+0=8)
B ≻ C ≡ 9 ≻ 8 ≡ (5+0+4= 9) ≻ (2+2+4=8)
C ≻ A ≡ 10 ≻ 7 ≡ (4+4+2=10) ≻ (0+5+2=7)

Figure 1. The Saari triangle with an example case (Saari, 2008).

Three-Candidate Elections Using Saari Triangles

http://demonstrations.wolfram.com/ThreeCandidateElectionsUsingSaariTriangles/

THE PLURALITY METHOD(S=0) THE ANTIPLURALITY METHOD(S=1) THE BORDA COUNT (S=0.5)



SEE THE PROCEDURE LINE!

VOTING AND MCDM – AN IMAGINARY EXAMPLE

"As candidates and voters in SC are put to stand for

alternatives and criteria in MCDM"

The three alternative websites for digital services:

i) DIGITAL WEBSITE A
ii) DIGITAL WEBSITE B
iii) DIGITAL WEBSITE C

which are ranked according to SIX CRITERIA:

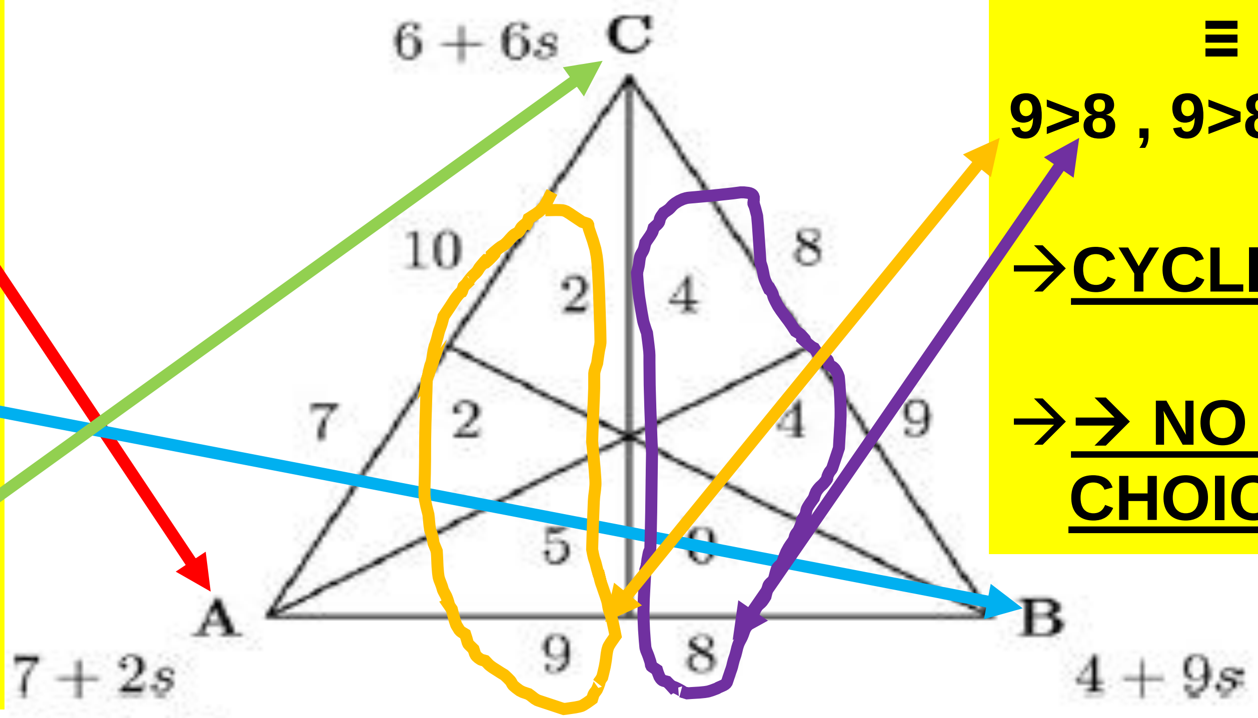
1) overall usability of digital services, with the weight 5 (=5votes)
2) flexibility between digital and face-to-face services, with the weight 4 (= 4 votes)
3) reliability of information, with the weight 4 (=4 votes)
4) interoperability of the information systems, with the weight 2 (=2 votes)
5) cost-effectiveness (of the services), with the weight 2 (= 2 votes)
6) many other unspecified criteria, with a zero weight (=0 votes)

WITH THE PLURALITY RULE ("VOTE FOR ONE") A WINS

WITH THE ANTI-PLURALITY RULE ("VOTE FOR TWO") B WINS

WITH THE BORDA COUNT (2,1,0) C WINS

WEBSITE [C]



(a) Profile.

<u>THE PAIRWISE RANKINGS LEAD TO A DEAD END</u>

A>B, B>C, C>A

≡

9>8 , 9>8 , 10>7

→<u>CYCLIC PREFERENCES</u>

→<u>→ NO RATIONAL CHOICE</u>

WEBSITE [A]

WHICH IS(ARE) THE BEST CHOICE(S)??

WEBSITE [B]

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