

Properties of the Hub&spoke Graph Structures

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Abstract. This paper presents the results obtained during the study of properties of hub-and-spoke structures widely used, for example, in logistics, obtained for different numbers of hubs generated using graph structures called alpha-cliques (or α -cliques), proposed by our team. We generated 100 weighted graphs of different sizes and connection numbers with fixed structures in which we searched for better hub-and-spoke structures. To find the hub-and-spoke structures discussed here in these graphs, we used a specialized evolutionary algorithm whose goal was to find the best possible structures of these graphs, based on the adopted quality criterion. As a result, we obtained experimental estimates of the number of hubs for this criterion depending on the size of the graphs under consideration. This could become the basis for theoretical analysis of the properties of such logistic structures, rather than simply calculating them for specific examples using relatively slow heuristic algorithms. This could become the basis for developing fast and efficient algorithms for generating hub-and-spoke structures for real-world cases, reflecting the fundamental rationality of the hub-and-spoke structure.

Keywords: hub-and-spoke, evolutionary algorithm, graph, hub-and-spoke profitability, hub number

1 Introduction

The hub-and-spoke (H&S) structures (the formal definition of it is presented in section 2) have been for quite some time suggested for given graphs of transport-related connections as the means of concentrating the flows of transported persons or goods. The benefits from the introduction of the hub-and-spoke structure have been widely recognized for several decades, but, in fact, they were rarely, if at all, subject to systematic and formally sound analysis, especially from the point of view of their actual profitability ([2], [10]). Since the problem of establishing the hub-and-spoke structure for a given graph of connections is not very precisely formulated (and may, in fact, have no solutions at all, especially if not "sufficiently strictly" formulated), it appears to be reasonable to study its certain general characteristics, especially regarding the broadly conceived profitability of the potential hub-and-spoke solutions. As discussed in our earlier work [6], the essential properties of such structure are highly dependent on the number of hubs. Thus, we decided to investigate the

influence of this number on the level of performance improvement that can be achieved using the hub-and-spoke structure.

The analysis we propose has become of special interest in the recent period, with the shift in the paradigms, ruling the transport network design, in particular – in the air transport. The dramatic increase in the market share of cheap operators, decrease in demand for the “jumbo-jets” (see, for instance, termination of production of the aircraft A380 in 2021), as well as the overall increase in demand, coupled with widening of its spatial spread, all this contributed to an erosion of the significance of the hub-and-spoke concept as the leading one, and to the quite general discussion of the potential new concept(s). A separate impact came from COVID-19 with the limitations on the numbers of passengers in the aircraft and in the surface facilities [4]. These circumstances definitely point out the necessity of a deeper analysis of (most) convenient and profitable structures, including the hub-and-spoke ones.

2 Basic definitions

Since we shall be modeling the entirety of the hub-and-spoke problem through the graph representation and then solving it as a problem defined on graphs, we start with the basic notions from graph theory, here given following [21].

A **graph** is a pair $G = (V, E)$, where V is a non-empty set of *vertices* and E is a set of *edges*. Each edge is a pair of vertices $\{v_1, v_2\}$ with $v_1 \neq v_2$.

A **degree** of a vertex is the number of edges to which this vertex belongs.

A **clique** (a *complete subgraph*) $Q = (V_q, E_q)$ in a graph $G = (V, E)$ is a graph such that $V_q \subseteq V$ and $E_q \subseteq E$ and either $\text{Card}(V_q) = 1$ or each pair of vertices $v_1, v_2 \in V_q$ fulfills the condition $\{v_1, v_2\} \in E_q$. ([3]). Each subgraph of a clique is a clique.

An **α -clique** ([6], [9], [15], [16], [18], [19]) can be defined as follows: let $A = (V', E')$ be a subgraph of a graph $G = (V, E)$, $V' \subseteq V, E' \subseteq E$, $k = \text{Card}(V')$ and let k_i be the number of vertices $v_j \in V'$ such that $\{v_i, v_j\} \in E'$.

1. For $k = 1$ the subgraph A of graph G is an **α -clique(α)**.

2. For $k > 1$ the subgraph A of graph G is an **α -clique(α)** if for all vertices $v_i \in V'$ the condition $\alpha = (k_i + 1)/k$ is fulfilled, where $\alpha \in (0, 1]$.

Further on we will use the notion of **α -clique** in the sense of **α -clique(α)** for an earlier established α . A subgraph of an **α -clique** may not be an **α -clique** for the established α .

A **hub-and-spoke** structure ([13], [14], [19]) (Fig. 1b) is a graph $H_s = (G_h \cup G_s, E)$, where the subset G_h corresponds to at least a connected graph (of hubs) with the relevant subset of the set E , and each vertex of the subset G_s (of spokes) has degree 1 and is connected exactly with one vertex from subset G_h , ([6], [7]).

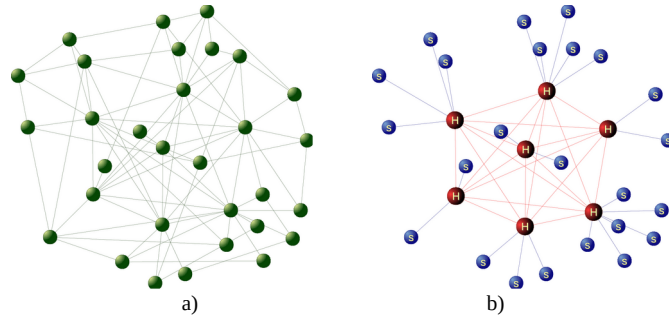


Fig. 1. a) A source graph and b) the hub-and-spoke structure obtained for this source graph

3 Generation of the analyzed graphs

In the study here reported we generated 100 graphs with imposed properties: a fixed hub-and-spoke structure hidden in the generated graphs and supplemented with the remaining edges to obtain complete graphs. In each case, the number of hubs and the number of spokes, the same for every hub, were predetermined. The method of generating such a graph was as follows. The number of hubs has been established. The hubs have been placed by putting each hub on a circle with a fixed radius, and with constant and equal angular distances between the subsequent hubs, as shown in Fig. 2. This is to ensure similar conditions for searching for a solution in the graphs under consideration.

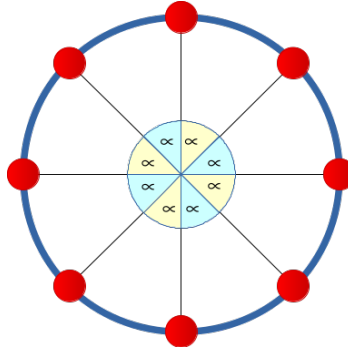


Fig. 2. The method for placing the hub nodes in the generated graphs

For the so defined structure of hubs, the spokes (corresponding to particular hubs) have been randomly placed inside the external belt with fixed inner and outer radii, as this is shown in Fig. 3.

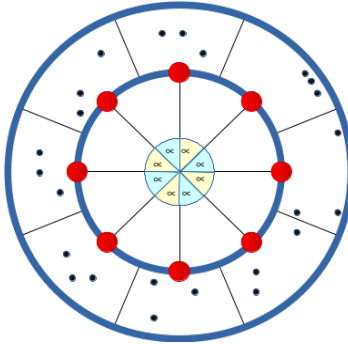


Fig. 3. Hubs with randomly placed spokes

4 The evolutionary method to obtain the hub-and-spoke structure

For the class of logistic and transportation problems that we are considering here, we propose to use evolutionary methods to obtain solutions. These methods can transform the transportation connections graph into an instance of the *hub-and-spoke* structure according to problem-specific restrictions. Evolutionary algorithms (EAs) are often used to solve hard graph problems, such as graph coloring, TSP, graph partitioning, maximum clique search, etc. ([1], [5], [20], [22]). The task of finding a graph covering with α -cliques, i.e. a certain variant of cliques, seems to have a similar level of difficulty thus, it is fully justified to also use this heuristic algorithm in the here considered graph transformation problem.

The standard evolutionary algorithm can be described by the scheme as shown below, but, actually, this simple scheme requires many problem specific modifications and additions in order to work efficiently for a definite problem ([11], [17]). The adjustment of the evolutionary algorithm for solving a given concrete problem requires a convenient encoding of solutions (easy to manage in computations), development of specialized genetic operators (random and knowledge-based) proper for the analyzed problem, and more efficient than simple roulette or even tournament selection methods, as well as, the fitness function to be optimized by the algorithm. Such a developed evolutionary method is often called a memetic algorithm [12].

The standard evolutionary algorithm

Input:

Input data

Output:

Output data

begin

Random initialization of the population of solutions.

while stop condition is not satisfied **do**

begin

Reproduction and modification of solutions using genetic operators

Valuation of obtained solutions

Selection of individuals for the next generation

end;

end;

A member of the population contains several data items including, in our case: a vector of hub nodes (numbers of the graph nodes, selected as hubs) with the lists of spokes attached to hubs, and, additionally, some other data, required by the specialized evolutionary algorithm: a vector of real numbers, describing the knowledge, concerning genetic operators and the index of the operator chosen to modify the solution in the current iteration. Also a set of dedicated genetic operators is prepared:

- mutation – exchange of randomly chosen nodes in different sets of spokes,
- relocation – moving a randomly chosen node to a different set of spokes, exchange of a randomly selected hub for a randomly selected spoke and
- concatenation – used only in the version of the algorithm with elimination of the zero-spoke hubs, it concatenates the deleted hub to the closest set of spokes.

In the here considered H&S problem the selected *hubs* should constitute together a complete subgraph, but when the connections between nodes are very sparse or are

determined as existing junction nodes (for instance: railway stations), it is admitted that the subgraph of hubs constitute an α -clique with α as high as possible or simply a connected graph (when the whole graph of connections is very sparse). In the case of weighted graphs, the weights of edges connecting hubs should have values as big as possible.

For the *hub-and-spoke* structure with an imposed number of hub nodes ([8], [19]) we use the quality function that promotes solutions with strong connections (weights) among hubs and between hubs and their spokes:

$$\max Q = \frac{1}{n} \sum_{i=1}^n \left(\sum_{l=i+1}^n w_{il} + \sum_{p=1}^{k_i} w_{ip} \right) \quad (1)$$

where: i, l – are the indices of hubs, n – is the imposed (or in case of eliminating the zero-spoke hubs – calculated by EA) number of hubs, k_i – is the number of spoke nodes (stops), corresponding to hub i in the considered graph⁵, w_{il} – weight of connection between the i -th and l -th hub, w_{ip} – weight of connection between the i -th hub and p -th node of this hub's subgraph of spokes.

In the here provided calculation examples, the weights are obtained as simple functions of distances (maximum Euclidean distance in the data set minus the distance between the particular pair of nodes).

The fitness function (1) maximizes the weight of connections between the selected stops – hubs ($\sum_{l=i+1}^n w_{il}$), as well as hubs and their spokes ($\sum_{p=1}^{k_i} w_{ip}$). It can be

proposed that this objective function reflects the rationality of the potential hub-and-spoke structure, implying, in particular, its better profitability.

The algorithm employed tries to find a better configuration, for the given, generated initial graph of connections and its weights of edges w_{ip} , for the consecutive (imposed) numbers of hubs (n). The search is performed in terms of (a) selection of hubs – among the nodes that both have been hubs in the initial graph and, possibly, also spokes, and (b) the assignment of spokes to these selected hubs.

This problem can be solved using a set of the specialized genetic operators, modifying the structure of a population member. When, for instance, the executed operator tries to move one node (spoke) to the subset, associated with another hub, it must first be checked for the connection with this new hub. If there is no such connection, the operation is canceled and no modifications are performed, so as to preserve the feasibility of the solution. As already mentioned, the set of genetic operators consists of:

- mutation – exchange of randomly chosen nodes in different sets of spokes;
- relocation of a randomly chosen node to a different set of spokes;
- exchange of a randomly selected hub for a randomly selected spoke.

5 Results of computer simulations

The graphs generated and results obtained using EA are illustrated in the following Table 1 for several examples.

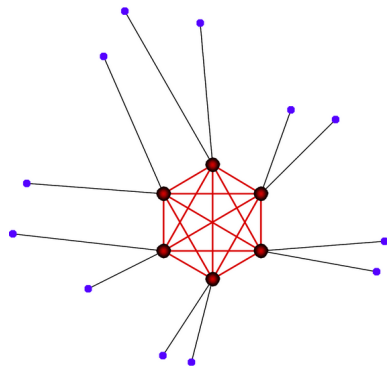
⁵It should be noticed that for the here considered connected graphs there is a simple relation

$k = n + \sum_{i=1}^n k_i = \sum_{i=1}^n (k_i + 1)$, where k is the number of nodes in the whole graph, which does not directly appear in (1), but all graph nodes are taken into account in quality function computations.

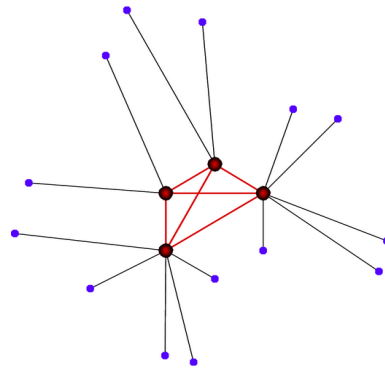
Table 1. Classes of graphs generated in the experiment (4 per each class)

Number of hubs	Number of spokes per hub	Inner radius	Outer radius	Number of hubs (h) proposed by EA	Approx. formula (2): $h = \lfloor \log_2 k \rfloor$
6	2	100	200	4	3
6	3	100	200	4	4
6	4	100	200	4	4
10	2	100	200	4	4
10	3	100	200	5	5
10	4	100	200	5	5
10	5	100	200	5	5
12	2	150	200	5	4
12	3	150	200	5	5
12	4	150	200	5	5
12	5	150	200	5	6
12	6	150	200	6	6
12	7	150	200	6	6
16	2	250	200	5	5
16	3	250	200	6	5
16	4	250	200	6	6
16	5	250	200	6	6
16	6	250	200	6	6
16	7	250	200	7	6
24	2	450	200	6	5
24	3	450	200	6	6
24	4	450	200	7	6
24	5	450	200	7	6
24	6	450	200	7	7
24	7	450	200	7	7

The examples of generated graphs and of the corresponding optimal solutions, obtained with the EA with respect to the function (1) and assumed $\log_2 n$ number of hubs, are shown in the following sequence of figures (Fig. 4ab – Fig. 16ab).

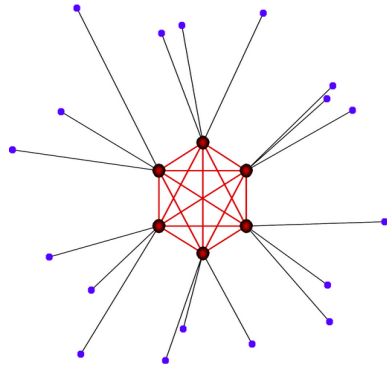


a)

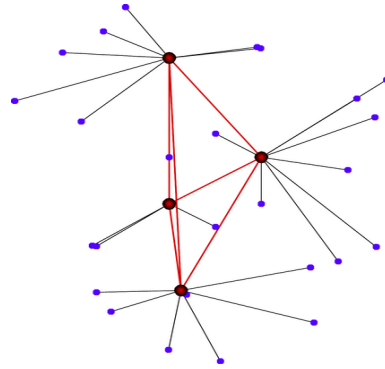


b)

Fig. 4a. A random graph with the fixed number of hubs⁶ - 6, and 2 spokes per hub, **Fig. 4b.** The H&S structure for the graph from Fig. 4a, obtained by EA.

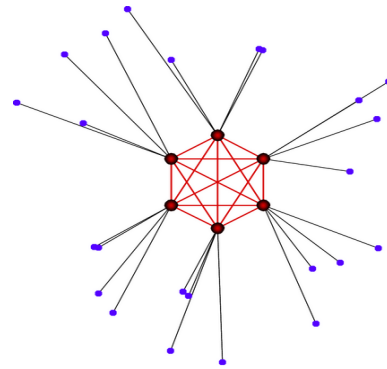


a)

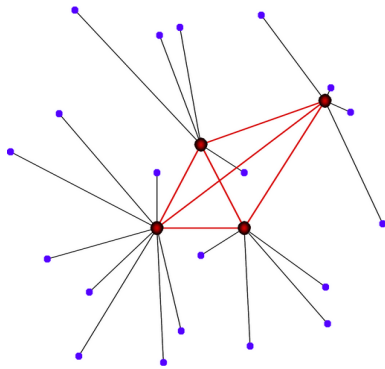


b)

Fig. 5a. A random graph with the fixed number of hubs - 6, and 3 spokes per hub, **Fig. 5b.** The H&S structure for the graph from Fig. 5a, obtained by EA.



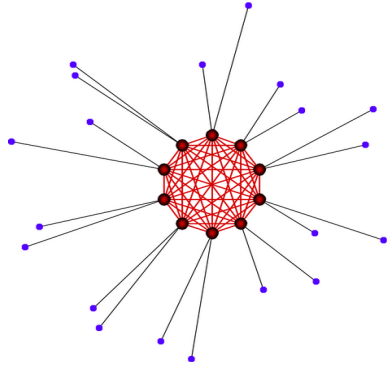
a)



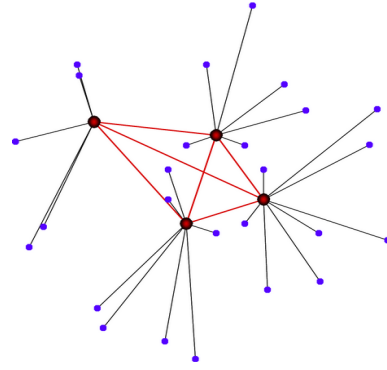
b)

Fig. 6a. A random graph with the fixed number of hubs - 6, and 4 spokes per hub, **Fig. 6b.** The H&S structure for the graph from Fig. 6a, obtained by EA.

⁶ Each of the presented graphs shows only the connections selected for the H&S structures.

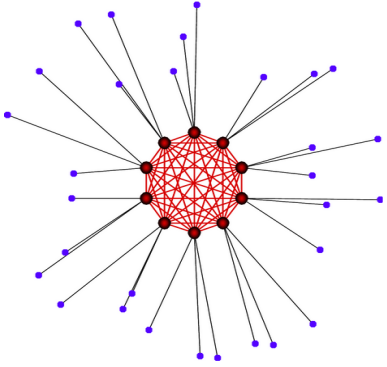


a)

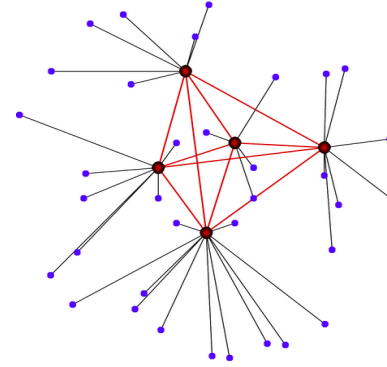


b)

Fig. 7a. A random graph with the fixed number of hubs - 10, and 2 spokes per hub, **Fig. 7b.** The H&S structure for the graph from Fig. 7a, obtained by EA.

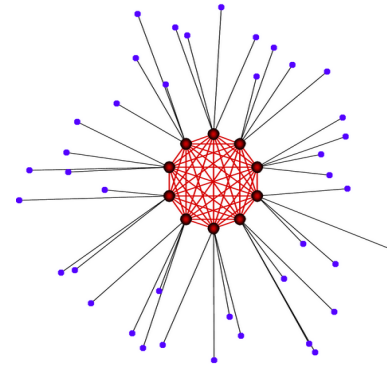


a)

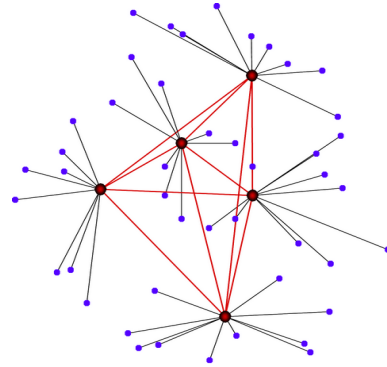


b)

Fig. 8a. A random graph with the fixed number of hubs - 10, and 3 spokes per hub, **Fig. 8b.** The H&S structure for the graph from Fig. 8a, obtained by EA.

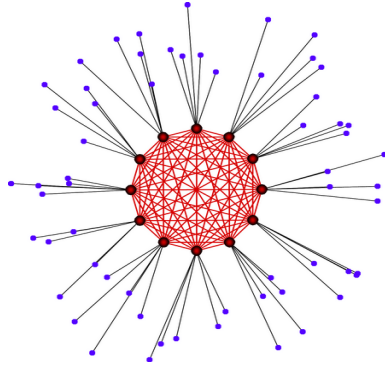


a)

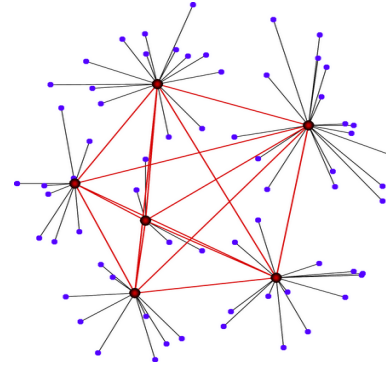


b)

Fig. 9a. A random graph with the fixed number of hubs - 10, and 4 spokes per hub, **Fig. 9b.** The H&S structure for the graph from Fig. 9a, obtained by EA.

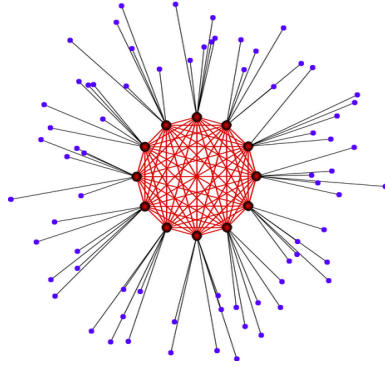


a)

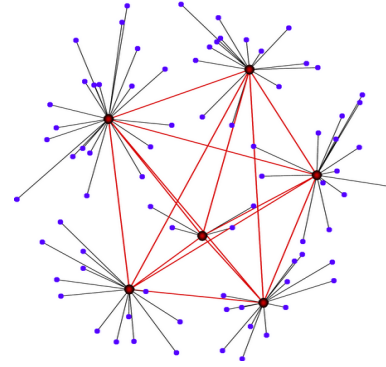


b)

Fig. 10a. A random graph with the fixed number of hubs - 12, and 4 spokes per hub, **Fig. 10b.** The H&S structure for the graph from Fig. 10a, obtained by EA.

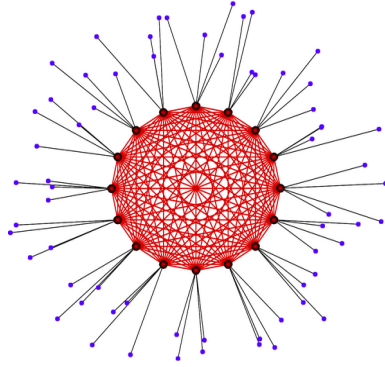


a)

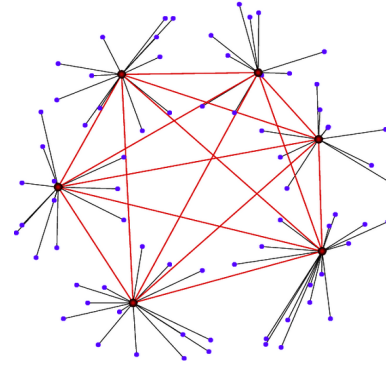


b)

Fig. 11a. A random graph with the fixed number of hubs - 12, and 6 spokes per hub, **Fig. 11b.** The H&S structure for the graph from Fig. 11a, obtained by EA.

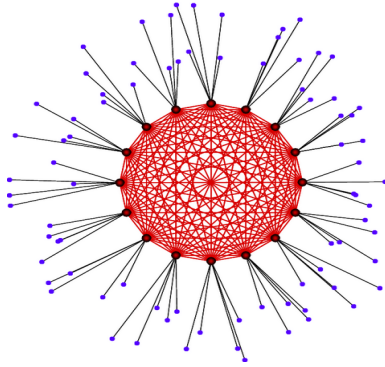


a)

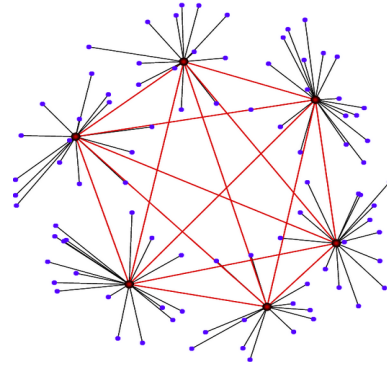


b)

Fig. 12a. A random graph with the fixed number of hubs - 16, and 4 spokes per hub, **Fig. 12b.** The H&S structure for the graph from Fig. 12a, obtained by EA.

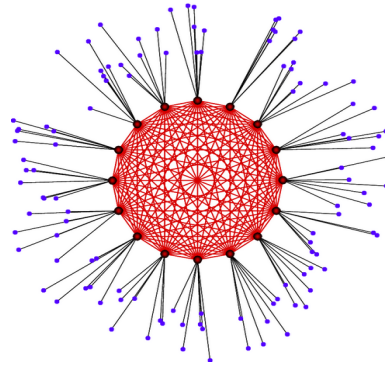


a)

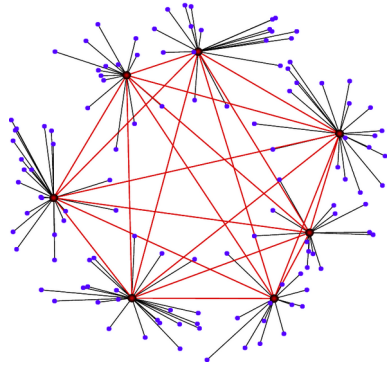


b)

Fig. 13a. A random graph with the fixed number of hubs - 16, and 5 spokes per hub, **Fig. 13b.** The H&S structure for the graph from Fig. 13a, obtained by EA.

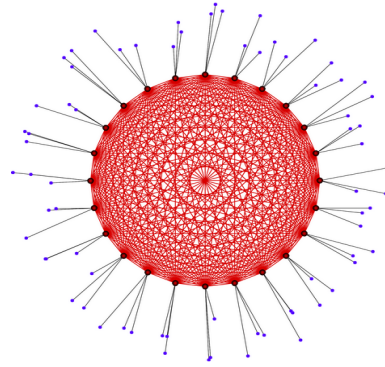


a)

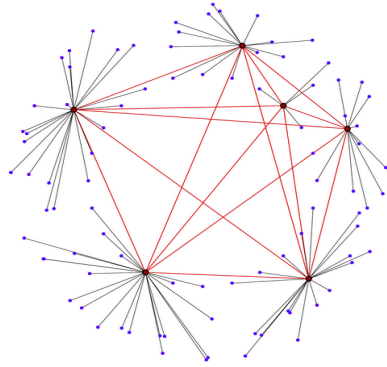


b)

Fig. 14a. A random graph with the fixed number of hubs - 16, and 7 spokes per hub, **Fig. 14b.** The H&S structure for the graph from Fig. 14a, obtained by EA.

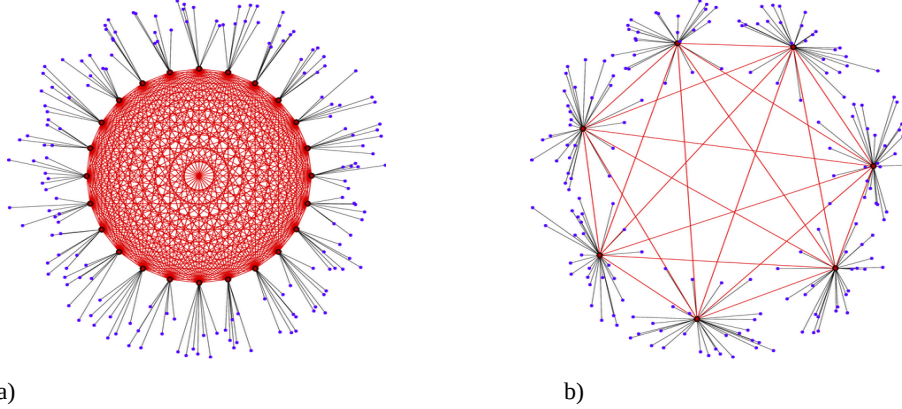


a)



b)

Fig. 15a. A random graph with the fixed number of hubs - 24, and 3 spokes per hub, **Fig. 15b.** The H&S structure for the graph from Fig. 15a, obtained by EA.



a) b)
Fig. 16a. A random graph with the fixed number of hubs - 24, and 7 spokes per hub, **Fig. 16b.** The H&S structure for the graph from Fig. 16a, obtained by EA.

Surprisingly for us, AE found completely different solutions, but it allowed us to draw some conclusions about the dependence of the number of hubs on the size of the graph. After many tests, for which we compared the obtained solutions and the initial, generated graphs, we were able to establish that the number of hubs for the class of problems here considered, which helps in finding good solutions, is approximately given by the formula (see results presented in Table 1):

$$h = \lfloor \log_2 k \rfloor \quad (2)$$

where k is the total number of graph nodes in the graph, i.e.

$$k = n + \sum_i k_i \quad (3)$$

and h is the sought, advantageous number of hubs, k_i – number of spoke nodes assigned to hub i .

We analyzed the obtained, supposedly optimal, structures. It looks like the logarithmic number of hubs in combination with our specialized evolutionary algorithm yields clearly better structures than the input structures (which were already the hub-and-spoke structures in themselves!). A lower number of hubs provides for a bigger concentration of flows, and so fewer big centers are needed for the efficient functioning of the entire logistic system. The respective values of the quality function (1), obtained from the experiments performed (averages of results for the particular classes of graphs from Table 1), are shown in the curves of Fig. 17.

The diagram of Fig. 17 illustrates, actually, four different cases: the values, obtained for the initial graphs (“Source”), the values, which were obtained from the direct application of EA to the initial graph (“EA”), i.e. with no change in the number of hubs, the solutions, obtained with application of formula (2) and the EA (“EA-log”), and, in addition, the results for the EA algorithm with elimination of hubs having at a given iteration no spokes (“EA-zero”). The results, illustrated in Fig. 17, were obtained for the maximization of function (1). This function, although representing some kind of rationality in designing the structure of the transport or logistic system, can be, naturally, subject to debate. This is insofar more true that there is no direct transformation between this function and some true economically based quality index, if it can at all be calculated, that is. In any case, one can imagine a slightly different, similarly rational form of the objective (fitness) function than (1), e.g. with the factor $1/n$ (division by the number of hubs) replaced by $1/k$ (division by the number of edges

in the output graph). If we maximize this function, the curves of Fig. 17 become those of Fig. 18.

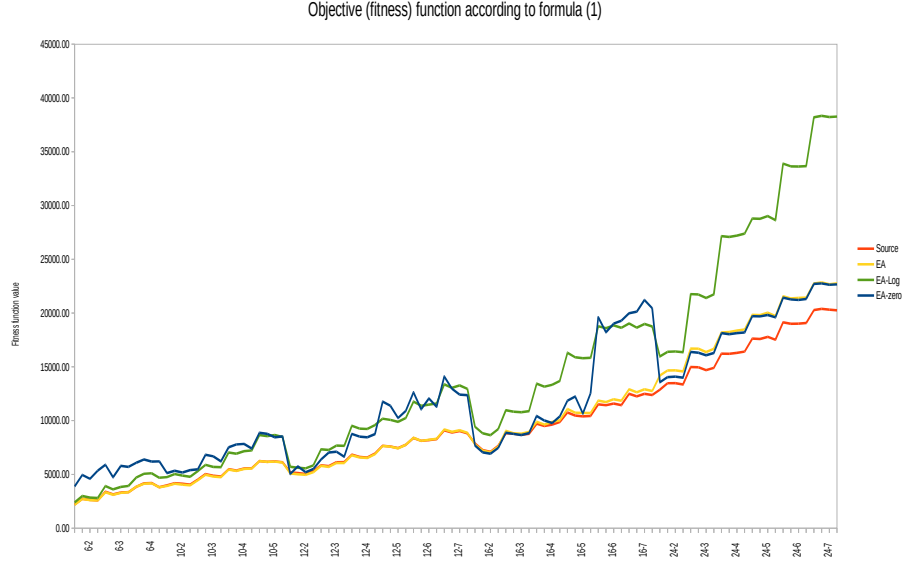


Fig. 17. The values of quality function (1) obtained for the initial graphs (“Source”), direct application of the evolutionary algorithm (“EA”), the EA performed for the number of hubs, given by formula (2) (“EA-Log”), and the EA with elimination of hubs having no spokes (“EA-zero”). The sequence of classes of initial graphs (horizontal axis) follows Tab. 1.

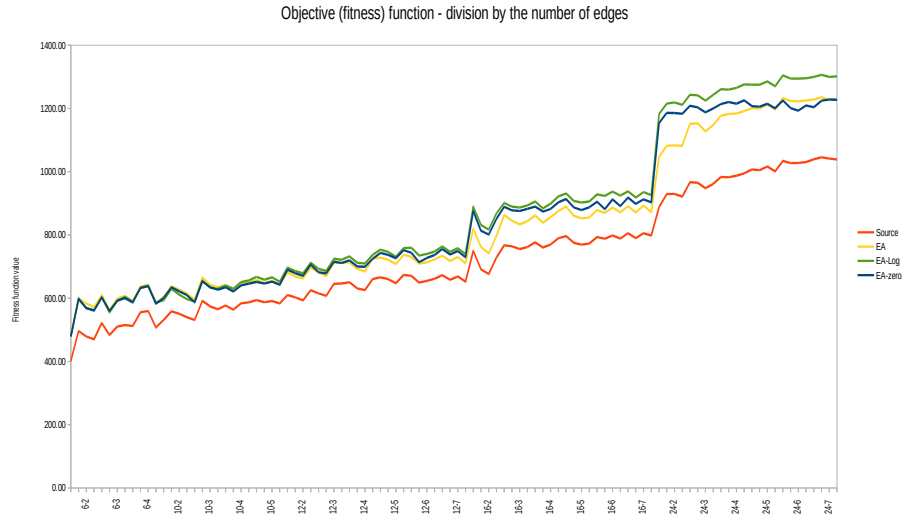


Fig. 18. The values of quality function with division by the number of edges obtained for the same instances as in Fig. 17. The sequence of classes of initial graphs (horizontal axis) follows Tab. 1.

The situation is different than before, even though the dominant solution remains that

based on formula (2), with, again, this domination increasing as the graph magnitudes and scopes of choice increase.

Finally, calculations were also performed for the objective (fitness) function that apparently lacks the rationality of the preceding two, namely – when formula (1) is devoid of the scaling division, i.e. we just maximize the overall throughput. This, naturally, does not entail any requirement on the lowered number of hubs or better utilization of existing edges, and the results are shown in Fig. 19. The lowered number of hubs leads to much worse results, as expected. The class of graphs, for which the empirical formula (2) is a good approximation, is defined by several parameters and characteristics, such as (I) existence of the explicit initial H&S structure; (II) equal distribution of spokes among the hubs in the initial structure; and (III) the range of graph magnitudes, hub proportions and graph densities. Concerning the latter, it can be easily concluded from Fig. 17 that the formula (2), coupled with the EA, yields the least of advantage – quite naturally – for the structures, for which the choice is the narrowest, i.e. the numbers of nodes are low, and the initial numbers of spokes per hub are also low. This implies one direction of potential future studies. Another such important potential direction of research is, of course, associated with the modeling of the situation at hand through appropriate objective function(s), as illustrated here.

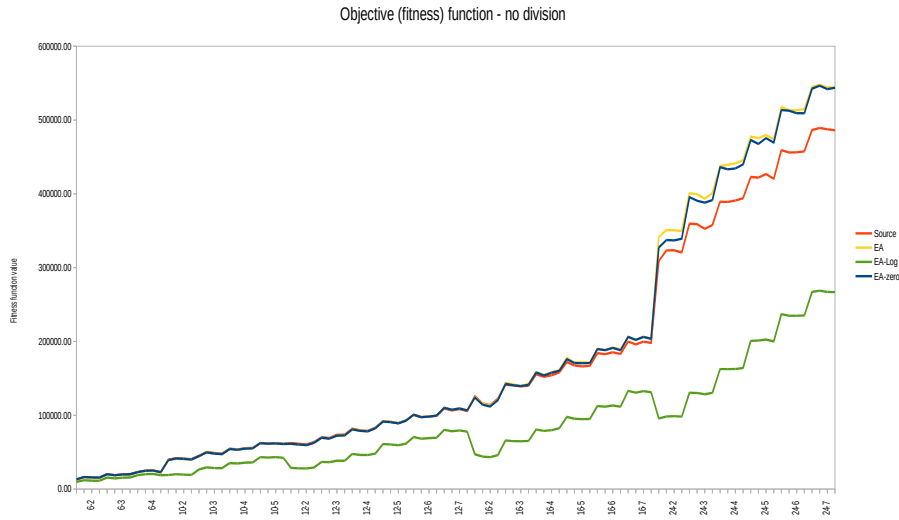


Fig. 19. The values of quality function (1) without the dividing factor, obtained for the same instances as in Fig. 17. The sequence of classes of initial graphs (horizontal axis) follows Tab. 1.

6. Conclusions

The logarithmic number of hub nodes, given by formula (2), looks like a good approximation for the proper number to obtain an essentially improved hub-and-spoke structure. It must be noted, though, that we tested this approach only for the initial complete hub graphs, and it should be further tested for a broader class of graphs, and also for a broader class of objective functions, or, more properly – for the objective functions reflecting better the actual issues in transport system design.

Still, two aspects ought to be strongly emphasized: (1) the fact that the initial graph is already built on a hub-and-spoke structure does not mean that an important improvement to this structure cannot be achieved; and (2) the range of this improvement appears to be truly astonishing if one looks at Fig. 17.

The approach presented appears as a promising methodology for supporting the design and planning of the public transport systems, as well as other logistic systems, where the hub-and-spoke structures can be applied without big costs, associated with the construction of new connections and nodes.

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