SUPPORTING THE PROCESS OF NEGOTIATIONS WITH THE USE OF MULTI-CRITERIA ANALYSIS

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Introduction

The presentation presents the application of multi-criteria optimization analysis to decision selection in the process of negotiations. Negotiations enable a decision to be reached in cases where the interests of the participants differ. Negotiations are carried out in order to reach a result more favorable than that which might have been achieved without negotiations.

The process of negotiations is modeled using the multi-criteria optimization problem. The method of decision selection is based on interactive selection of certain proposals for solutions, i.e. the algorithm requires the reaction of parties during this process. The parties submit their proposals concerning the subjects of the negotiations. These proposals comprise parameters of the multi-criteria optimization problem; thus the problem is solved. Then, the parties evaluate the solution, accepting or rejecting it. In the latter case, the parties submit new proposals with new values of parameters and the problem is solved again for these new parameters.

Modeling of the negotiatoins process

The negotiation process is modeled as an interactive decision-making process. Each party presents its proposals for solutions. The negotiation process then consists of seeking a common decision which reconciles the interests of both parties.

The following are given:

party 1 and party 2 are the parties in the negotiations;

n is the number of subjects for the negotiations;

$x \in X_0$ is a solution: a decision to which the parties are to agree, belonging to the set of

feasible decisions $X_0 \subset R^n$, $x = (x_1, x_2, \ldots, x_n)$; each coordinate $x_i, i = 1, \ldots, n$

defines the $i-th$ subject of negotiations;

$f1 : X_0 \to R^1$ is the evaluation function of a decision x by party 1;

$f2 : X_0 \to R^1$ is the evaluation function of a decision x by party 2.

The negotiation process is considered a problem of multi-criteria optimization with the function of purpose $f = (f1, f2)$:

$$\max_x \{(f1(x), f2(x)) : \qquad x \in X_0\}$$

where:

X_0 is the set of feasible decisions;

$x \in X_0$ is a vector of decision variables;

$f = (f1, f2)$ is the vector function which maps the decision space X_0 into the evaluation space $Y_0 \subseteq R^2$.

The multi-criteria optimization model (1) can be rewritten in an equivalent form in the space of achievement vectors. Consider the following problem:

$$\max_{x}\{(y_1, y_2): \qquad y \in Y_0\},$$

where:

$x \in X_0$ denotes a vector of decision variables;

$y = (y_1, y_2)$ is the achievement vector; the first coordinate is the evaluation criteria of a solution by party 1, the second coordinate is the evaluation criteria of a solution by party 2;

$\backslash Y_0 = f(X_0)$ is the set of achievement vectors

An equitably efficient decision

The solution in the negotiation process should satisfy certain properties that the parties accept as reasonable. The solution should be:

- an optimal solution in the sense of Pareto, i.e., such that it would be impossible to improve the solution for one party without making the solution worse for the other party;
- an anonymity solution, i.e., one that should not depend on the way the parties are numbered, so that neither is more important; the parties are treated in the same way in the sense that the solution does not depend on the name of, or other factors specific to, a given party;
- an equalizing solution, i.e., an evaluation vector characterized by lesser variation in terms of coordinates of evaluation is preferred in comparison to a vector with the same number of, but with a greater level of diversity of, coordinates;
- the solution should take the strength of the parties to the negotiations into account.

A decision, which satisfies these conditions is an equitably efficient decision. This is Pareto-optimal decision which satisfies additional conditions – the anonymity property and the transfer principle.

Nondominated solutions (optimum Pareto) are defined as follows:

$$\hat{Y}_0 = \{\hat{y} \in Y_0 : (\hat{y} + \widetilde{D}) \cap Y_0 = \varnothing \}$$

where:

$Y_0 = f(X_0)$ is the set of achievement vectors,

$\widetilde{D} = D \setminus \{0\}$ is a positive cone without the top. As a positive cone, it can be adopted $\widetilde{D} = R_+^2$

In multi-criteria problem (1), which is used to select a decision in the negotiation process, the relation of preferences should satisfy additional properties: the anonymity property and the transfer principle.
This preference relation is called an anonymous relation if, for every assessments $y=(y_1, y_2) \in R^2$ and for any permutation P of the set $\{1, 2\}$, the following property holds:

$$(y_{P(1)}, y_{P(2)}) \approx (y_1, y_2)$$

No distinction is made between results that differ in their arrangement. Evaluation vectors with the same coordinates, but stated in a different manner, are identified

The relationship of preferences should satisfy the principle of transfer provided that the following condition is satisfied:

for the evaluation vector $y = (y_1, y_2) \in R^2$:

$$y_{i'} > y_{i''} \Rightarrow y - \varepsilon \cdot e_{i'} + \varepsilon \cdot e_{i''} \succ y \ \text{ for } 0 < y_{i'} - y_{i''} < \varepsilon$$

Equalizing transfer is a slight deterioration of a superior coordinate of the evaluation vector and the simultaneous improvement of an inferior coordinate, yielding an evaluation vector which is strictly preferred over the initial evaluation vector

A nondominated vector satisfying the anonymity property and the axiom of equalizing transfer is called a equitably nondominated vector. The decision $\hat{x} \in X_0$ is called an equitably efficient decision if the corresponding evaluation vector $\hat{y} = f(\hat{x})$ is an equitably nondominated vector.

The relation of equitable domination can be expressed as the relation of inequality for cumulative, ordered evaluation vectors. This relation can be determined with the use of transformation $\overline{T} : R^2 \rightarrow R^2$, which accumulates coordinates in decreasing order in the evaluation vector.

The transformation $\overline{T} : R^2 \rightarrow R^2$ is defined as follows :

$$\overline{T}_i(y) = \sum_{l=1}^{2} T_l(y) \quad \text{for } i = 1, 2$$

where:

$T(y)$ is the vector with decreasing ordered coordinates of the vector y, i.e. $T(y) = (T_1(y), T_2(y))$, where $T_1(y) \leq T_2(y)$ and there is a permutation P of the set $\{1, 2\}$, such that $T_i(y) = y_{P(i)}$ for $i = 1, 2$.

The evaluation vector y^1 dominates the vector y^2 in an equitable manner if the following condition is satisfied:

$$y^1 \succ_e y^2 \Leftrightarrow \overline{T}(y^1) \geq \overline{T}(y^2)$$

Scalarization of the problem

For the determination of equitably efficient solutions of multi-criteria problem, a specific multi-criteria problem is solved. This is a problem with the vector function of the cumulative, ordered evaluation vectors, i.e. the following problem:

$$\max_{x}\{(\overline{T}_1(y), \overline{T}_2(y)): \qquad y \in Y_0\}$$

where:

$y = (y_1, y_2)$ is an evaluation vector;

$\overline{T}(y) = (\overline{T}_1(y), \overline{T}_2(y))$ is a cumulative, ordered evaluation vector,

Y_0 is the set of achievable evaluation vectors.

An efficient solution of multi-criteria optimization problem is an equitably efficient solution of multi-criteria problem.

The scalarizing function is as follows:

$$s(y,\bar{y}) = \min_{1 \le i \le 2} (\overline{T}_i(y) - \overline{T}_i(\bar{y})) + \varepsilon \cdot \sum_{i=1}^{2} (\overline{T}_i(y) - \overline{T}_i(\bar{y}))$$

where:

$y = (y_1, y_2)$ is an evaluation vector;

$\overline{T}(y) = (\overline{T}_1(y), \overline{T}_2(y))$ is a cumulative, ordered evaluation vector;

$\bar{y} = (\bar{y}_1, \bar{y}_2)$ is a vector of aspiration levels;

$T(\bar{y}) = (T_1(\bar{y}), T_2(\bar{y}))$ is cumulative, ordered vector of aspiration levels;

ε is an arbitrary small, positive adjustment parameter

Such a scalarizing function is called the function of achievement. The aim is to find a solution that approaches the specific requirements, i.e., the aspiration levels, as closely as possible.

Set of negotiations

Before starting the negotiations, parties should consider what result they will achieve if negotiations are unsuccessful: the *status quo* point. This point is the result which can be achieved by each party without negotiations with the other party. If the parties can achieve the result $ys = (y1s, y2s)$ without negotiations (i.e. part 1 can achieve the result $y1s$, part 2 the result $y2s$), neither party will agree to an inferior result. During negotiations, parties want to improve the solution in relation to this point. The *status quo* point determines the strength of the parties in the negotiations and their impact on the result

The set of negotiations is a collection of equitably dominated evaluation values dominating the status quo point.
The set of negotiations is as follows:

$$B(\hat{Y}_{Oe}, ys) = \{\hat{y} = (\hat{y}1, \hat{y}2) \in \hat{Y}_{Oe} \wedge \hat{y}1 \geq y1s \wedge \hat{y}2 \geq y2s\}$$

where:

$\hat{y} = (\hat{y}1, \hat{y}2) \in \hat{Y}_{0e}$ is the equitably nondominated vector;

$ys = (y1s, y2s)$ is the status quo point - the result, which can be achieved by both parties without agreement.

Method of selection the decision

The method of decision selection is as follows:

1. The initial arrangements.
2. Iterative algorithm—proposals for further decisions:
   2.1. Interaction with the system—parties define their proposals for individual subjects of negotiations as aspirations levels $\overline{y}1$ and $\overline{y}2$.
   2.2. Calculations—identifying another solution from the set of negotiations, $\hat{y} = (\hat{y}1, \hat{y}2) \in B(\hat{Y}_{0e}, ys)$.
   2.3. Evaluation of the obtained solutions $\hat{y} = (\hat{y}1, \hat{y}2)$ —the parties may or may not accept the solution. If not, the parties submit new proposals, providing new values for their aspiration levels $\overline{y}1$ and $\overline{y}2$, and a new solution is determined (see sec. 2.2).
3. Determination of the decision that meets the requirements of both parties.

An example of negotiatoins

The negotiation problem is as follows:

part 1 and part 2 are the parties attending the negotiations;

$n = 2$ is the number of subjects to be negotiated;

$x = (x_1, x_2) \in X_0$ is a decision to which the parties are to agree and which belongs to the set of feasible decisions $X_0 \subset R^2$,

$$X_0 = \{(x_1, x_2) \in R^2 : -2 \cdot x_1 + 3 \cdot x_2 \le 27, \quad 6 \cdot x_1 + 7 \cdot x_2 \le 175, \quad 0 \le x_1 \le 21, \; 0 \le x_2 \le 13\}$$ is

the set of feasible decisions;

$f1 : X \to R^1 \; f1(x) = \frac{20}{21} x_1 - \frac{2}{3} x_2$ is an evaluation function of decision x by party 1;

$f2 : X \to R^2 \; f2(x) = -\frac{4}{21} x_1 + \frac{2}{3} x_2$ is an evaluation function of decision x by party 2;

$ys = (ys1, ys2) = (10, 1)$ is the status quo point.

As a first step of the multi-criteria analysis, a single-criterion optimization of the evaluation function of each party is performed

Matrix of the implementation of goals with the utopia vector

Optimized criterion	Solution $y1$	$y2$
Evaluation by party 1 $y1$	20	-4
Evaluation by party 2 $y2$	-2.85	7.52
Utopia vector	20	7.52

Interactive analysis of the search for a solution

Iteration	Evaluation of party 1 $y1$	Evaluation of party 2 $y2$
1. Aspiration point $\overline{y}$	20	7.52
Solution $\hat{y}$	15.33	0.66
2. Aspiration point $\overline{y}$	15	7
Solution $\hat{y}$	14.99	0.83
3. Aspiration point $\overline{y}$	14	6,5
Solution $\hat{y}$	13.33	1.33
4. Aspiration point $\overline{y}$	13	6.5
Solution $\hat{y}$	12,99	1.83
5. Aspiration point $\overline{y}$	12	5
Solution $\hat{y}$	11.99	2.33

For iteration 5 the relevant decisions are as follows: $\hat{x}^5 = (18.81,\ 8.88)$

Conclusions

The presentation presents a method of modeling a process of negotiations in the form of a multi-criteria optimization problem, a process used to support the decision selection. Modeling the negotiation process as a multi-criteria optimization problem enables us to create variants of the decision and to track their consequences.

The method of interactive analysis, based on the reference point, is applied to a multi-criteria problem with a cumulative, ordered evaluation vector. This enables us to arrive at solutions which are tailored to the parties' preferences.

This procedure does not determine the final solution, but supports and teaches the parties about the specific negotiation problem. The final decision is to be made by the parties involved in the negotiations.

Thank you all for listening.