

# Short- and Long-term Objectives of an Enterprise. Bi-criteria Analysis

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# Introduction

How to reconcile different interests of different actors involved that have different objectives?

The answer is crucial for the control of the economic processes within the firm and in the negotiations

Short-term and long-term model of the firm consisting of:

**model of production** (contrary to the neoclassical production theory our model accounts for non-competitive environment such as in the case of the monopoly, oligopoly, network monopoly)

**model of production costs** (accounts for diversity of the components of the fixed costs)

**formal goals**, often conflicting, of the firms and actors (owners, employees, management)

- profit maximization (at least statutory goals in the case of firms acting in both the competitive and non-competitive environments),
- income maximization (family firms without hired labor, cooperatives, employees motivated by the system of incentives)
- two-criteria optimization

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# Model of production

Non-neoclassical production function

$$Q = F(K, L)$$

$K$  – capital, fixed assets

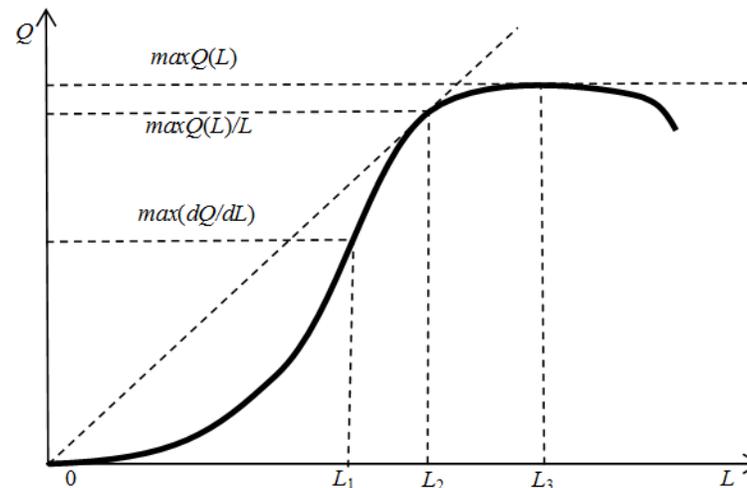
$L$  – labor

$Q$  – gross output

The following conditions are satisfied:

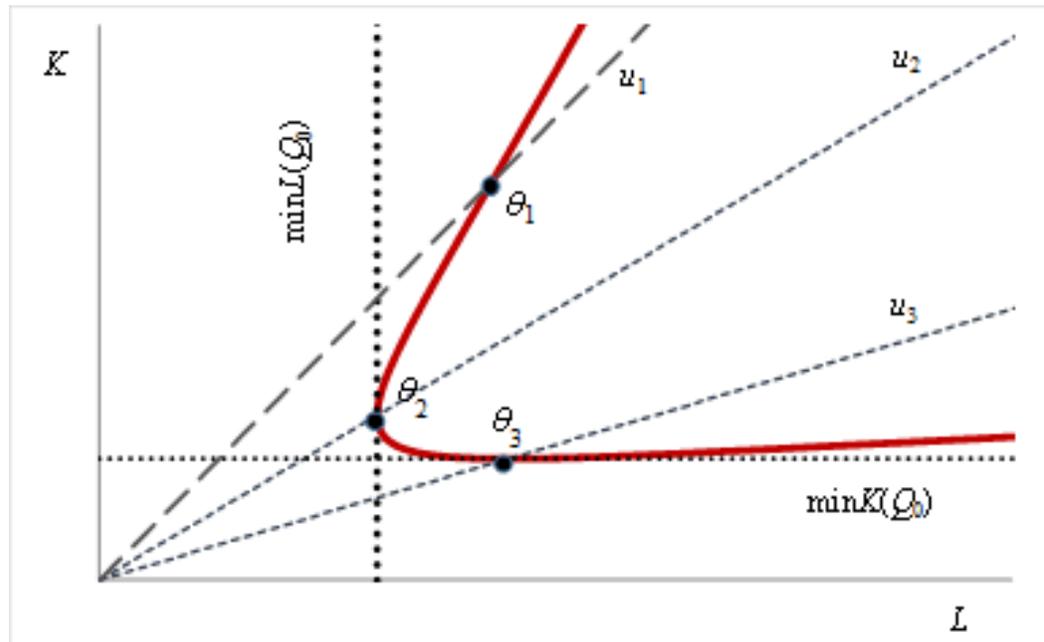
$$F(0,0) = 0; F(K, 0)=0; F(0, L) = 0.$$

Short-term analysis,  $K=K_0=const.$ , constant technology and management system, **Labor** is the only variable



# Model of production

Long-term analysis, constant: technology, management system, two variables: capital  $K$  and labor  $L$



Contour line of the non-neoclassical production function

Distinctive properties: Substitution of capital and labor is restricted to area between lines  $u_2$  and  $u_3$  (unlike neoclassical function where capital-labor substitution is unlimited)

# Model of production costs

Fixed costs

$$FC(Q) = \delta p_K K_0 + FC_0$$

where:  $FC_0$  - fixed costs, part unrelated to the fixed assets!

$\delta p_K K_0$  - amortization of the fixed assets

Variable cost

$$VC(Q) = wL + p_m q Q$$

Total cost

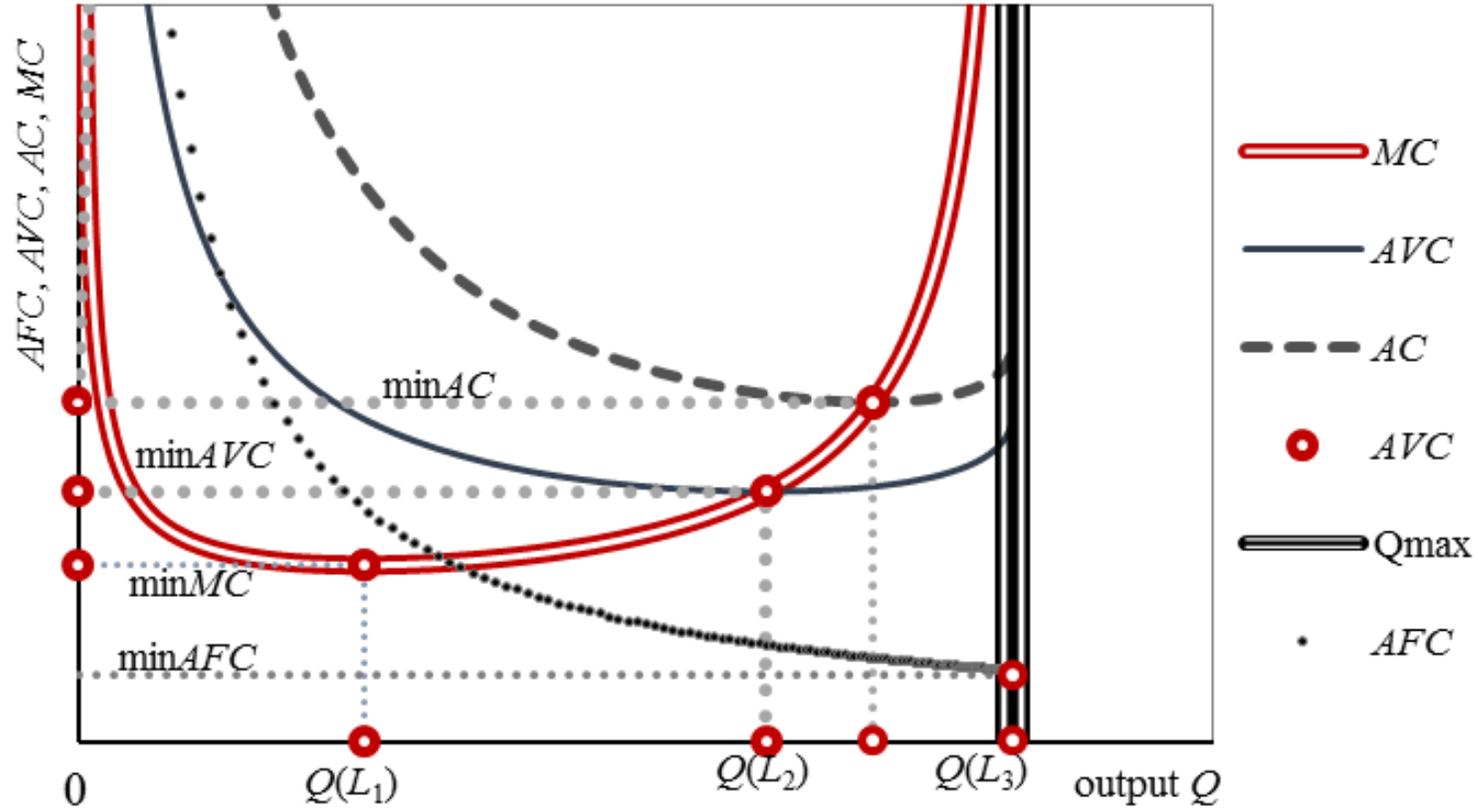
$$C(Q) = wL + p_m q Q + \delta p_K K_0 + FC_0$$

The above formula enable determination of the average costs  $AC$ ,  $AVC$  and  $AFC$

# Model of production costs

Average costs  $AC$ ,  $AFC$ ,  $AVC$  and marginal cost.

This handbook compatibility of costs and output is unavailable using the neoclassical production function!



# Environment, demand and objectives

## Environment

Revenue  $R(Q)$ :  $R(Q) = Q \cdot P(Q)$ , ( $P(Q)$  – price obtained from the sale of the  $Q$  units of output)

Model of perfect competition  $P(Q)=p=const. \frac{dP}{dQ} = 0$

Model of imperfect competition  $\frac{dP}{dQ} < 0$  (an increase of output causes decrease of price)

Market equilibrium ( $D$  – demand):  $Q=D$

Demand:  $P(Q) = P(D) = a_0 \exp(-a_1 Q)$ ;  $a_0, a_1 > 0$ .

Objectives of firms (differ due to the differences in the legal form of the ownership)

**Profit  $\pi(Q)$  maximization** (corporations, firms employing hired labor):

$$\pi(Q) = R(Q) - C(Q)$$

**Income  $INC(Q)$  maximization** (small family firms without hired labor, cooperatives, etc)

$$INC(Q) = R(Q) - [C(Q) - wL] = P(Q) \cdot Q - p_m \cdot q \cdot Q - \delta p_k K_0 - FC_0$$

# Objectives

Objectives differ not only between firms, but also within them.

There coexist different interests within a firm. In the simplest case there are distinct interests: that of the owners and the management who, for example, pursue statutory profit maximization, and that of the workers who pursue income maximization (when wages depend on the amount of output, this criterion is equivalent to the output maximization).

Solution of this conflict of interests requires determination of single-criterion optima of each group of interest, as well as bi-criteria optimization. Acquaintance of these optima is necessary in the negotiations aimed at establishment of the internal equilibrium of a firm.

## Bi-criteria analysis and numerical results

- We consider a firm in which decisions are made taking into account two groups of interests.
- The groups of interest have different goals: one pursues the income maximization  $INC(Q)$ , and the other pursues profit maximization  $\pi(Q)$ .
- The decision-making problem can be described as the vector bi-criteria optimization problem (with respect to the inputs of capital  $K$  and labor  $L$ ) shown below:

$$V_{\max} [INC(Q), \pi(Q)],$$

The problem is solved with respect to the capital and labor treated as variables. Requirements of positive values of the variables have to be satisfied:

$$K, L \geq 0, \text{ and } F(0,0) = 0, F(K,0) = 0, F(0,L) = 0.$$

Let  $\Omega$  denote the set of all admissible values of variables  $K, L$  satisfying the above conditions.

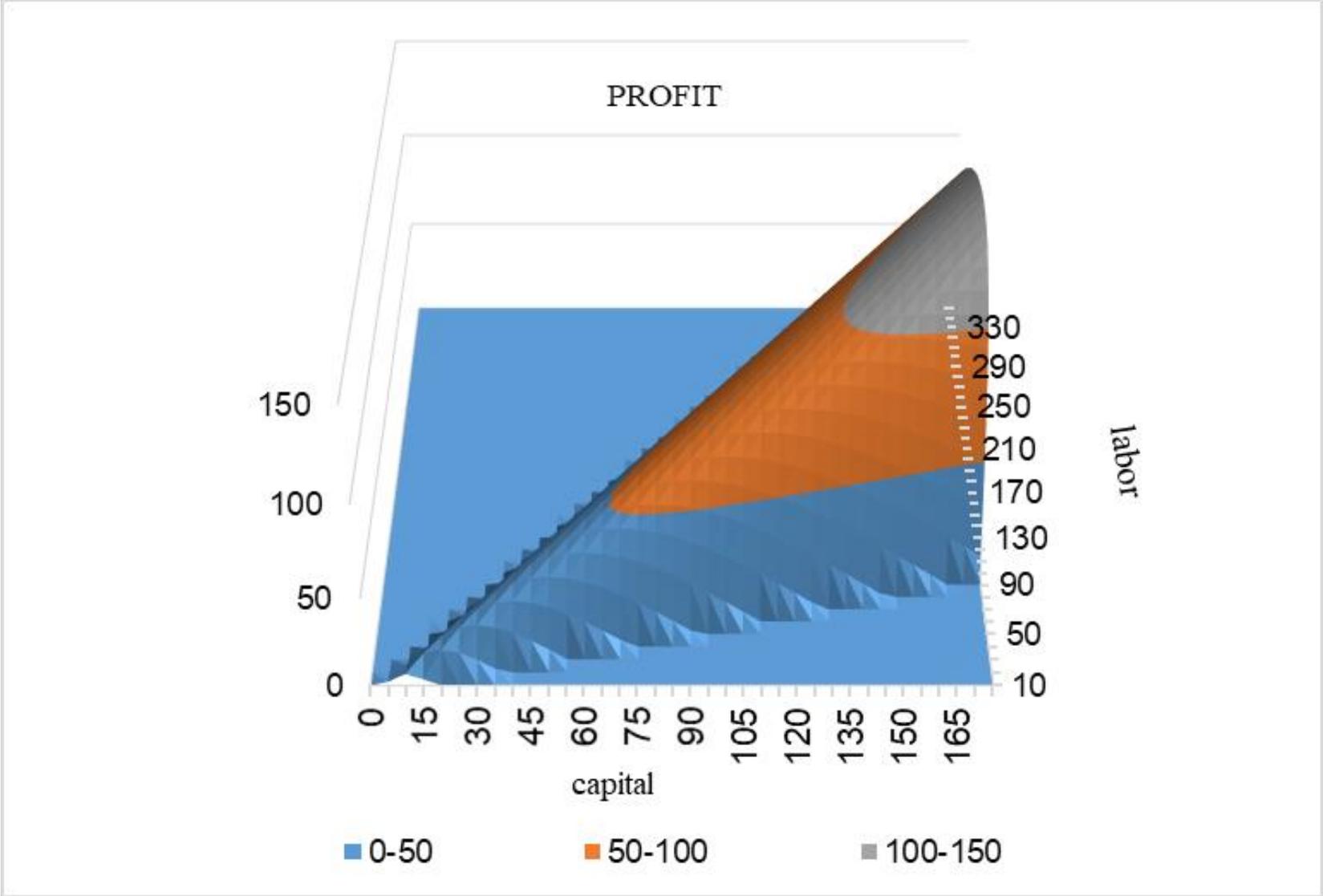
Operator  $V_{\max}$  means that we are looking for pairs of variables  $(K,L)^N \in \Omega$ , such that there does not exist any pair  $(K,L)$ , which would improve both criteria at the same time:  $INC$  and  $\pi$ .

The pairs  $(K,L)^N$  satisfying this condition are called non-dominated or Pareto-optimal in set  $\Omega$ .

In a general case there exist many Pareto-optimal solutions of the problem (30), including solutions maximizing only the income, or only the profit, but also intermediate ones, which can be negotiated while striving after a consensus.

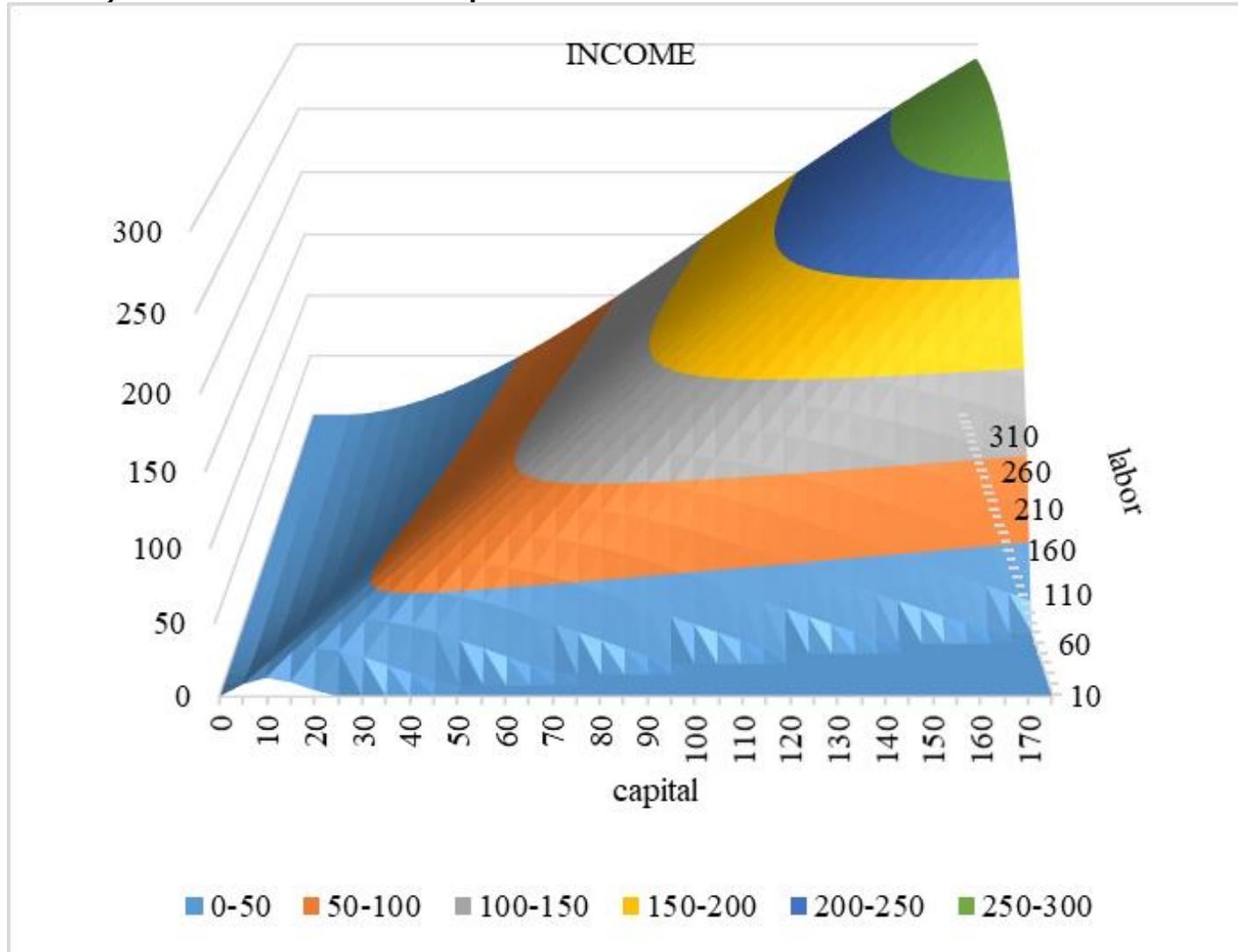
# Model of perfect competition

The dependency of profit on capital and labor



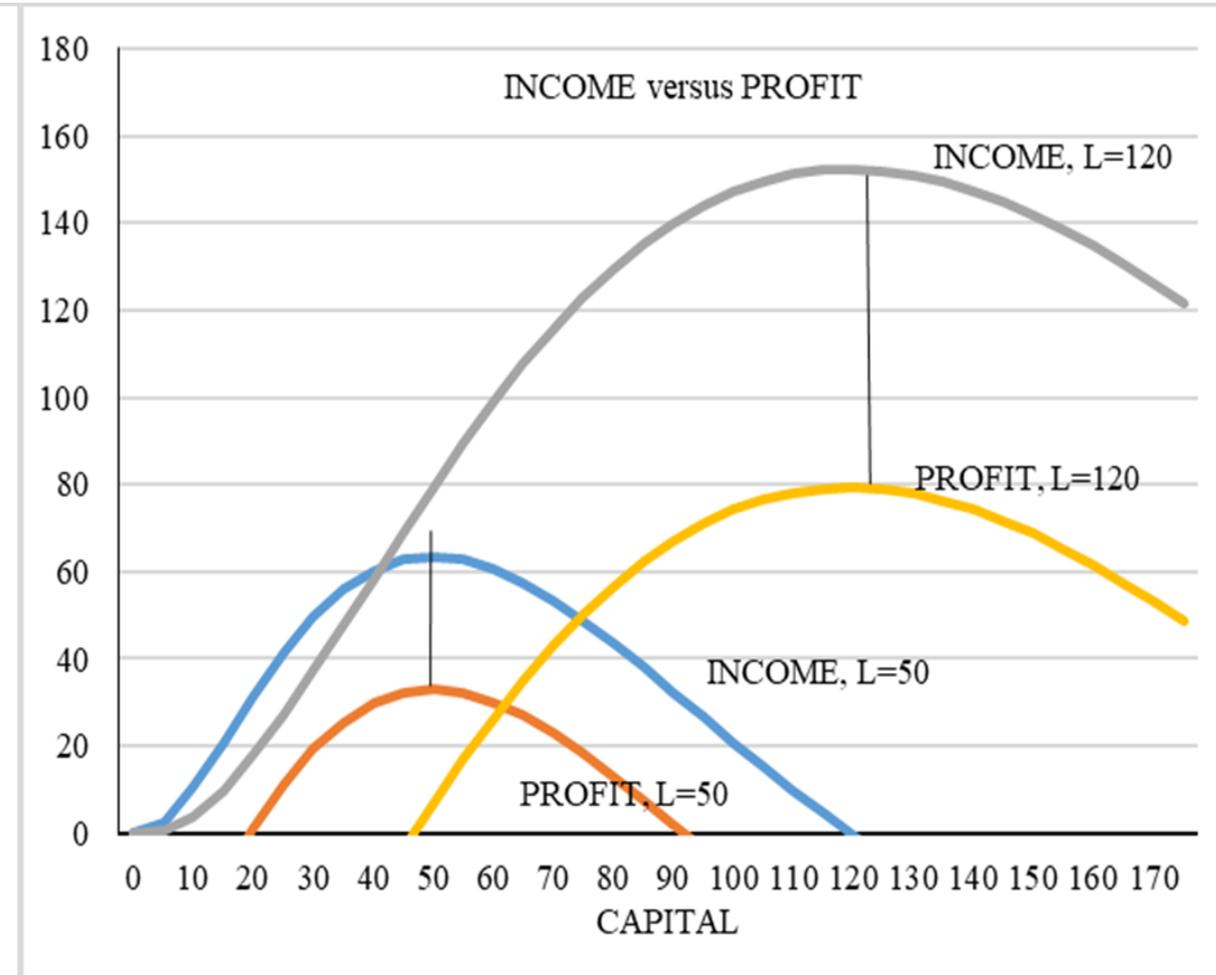
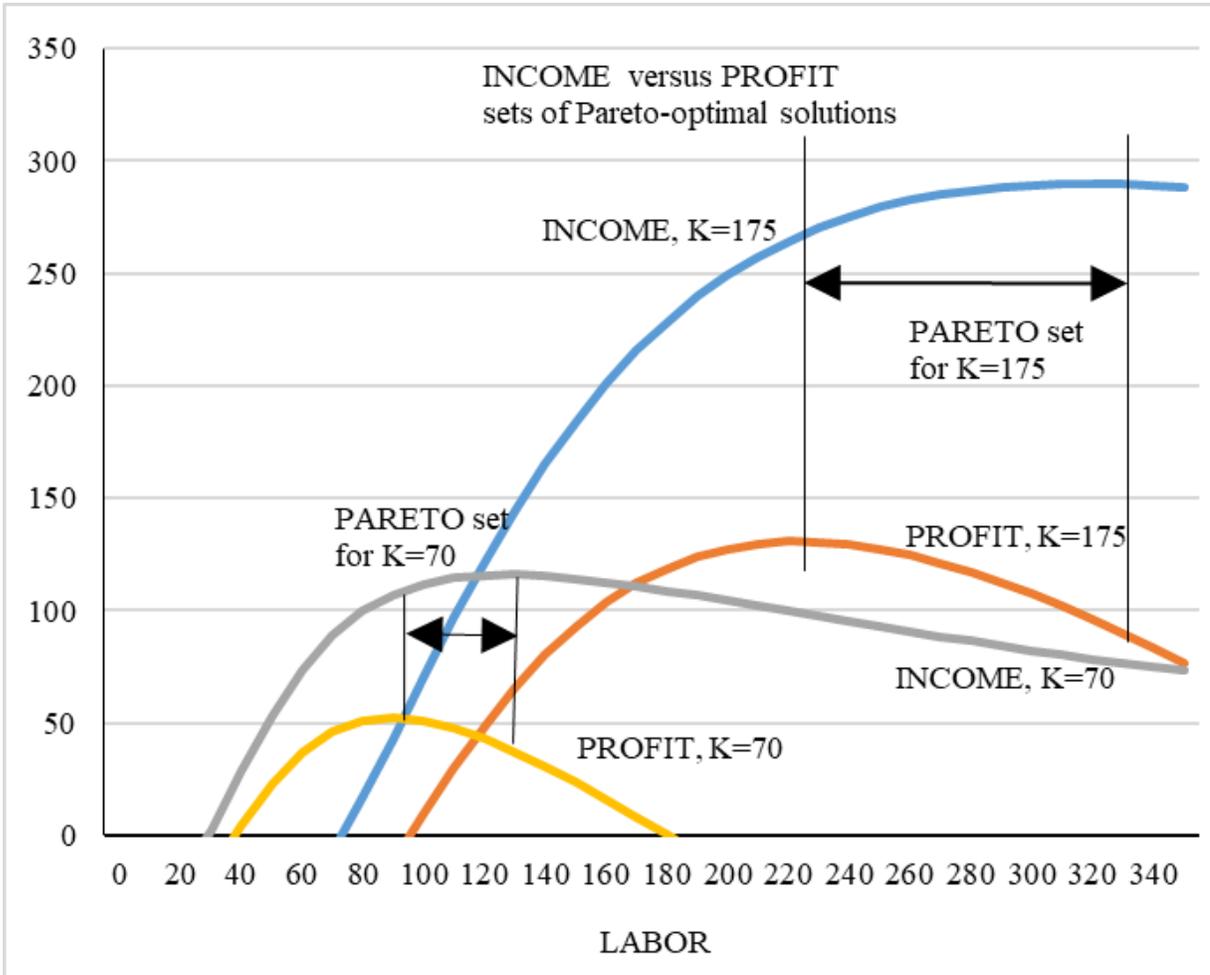
# Model of perfect competition

The dependency of income on capital and labor



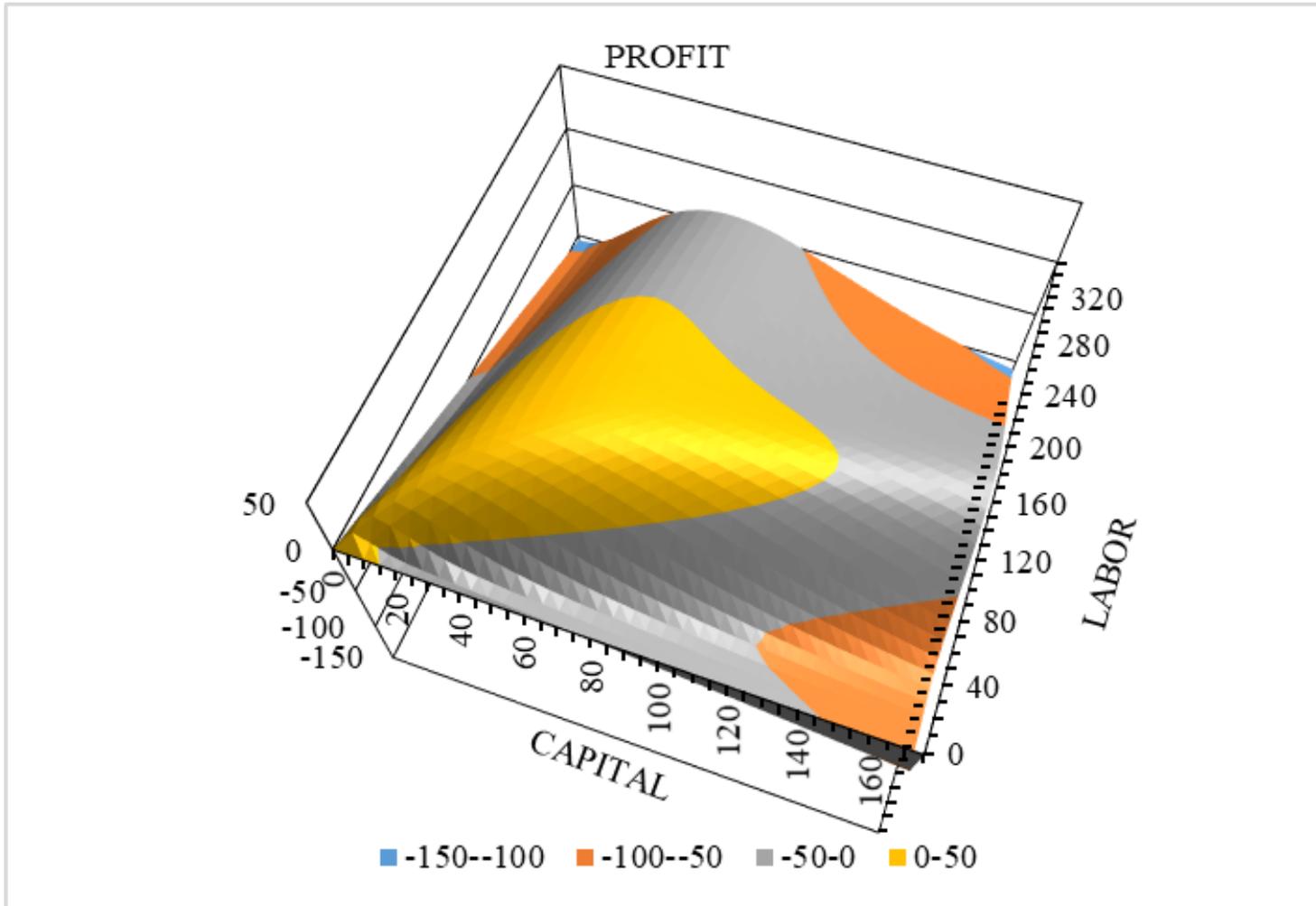
# Model of perfect competition

The income versus profit;  
dependency on labor for different values of capital & dependency on capital for different  
values of labor



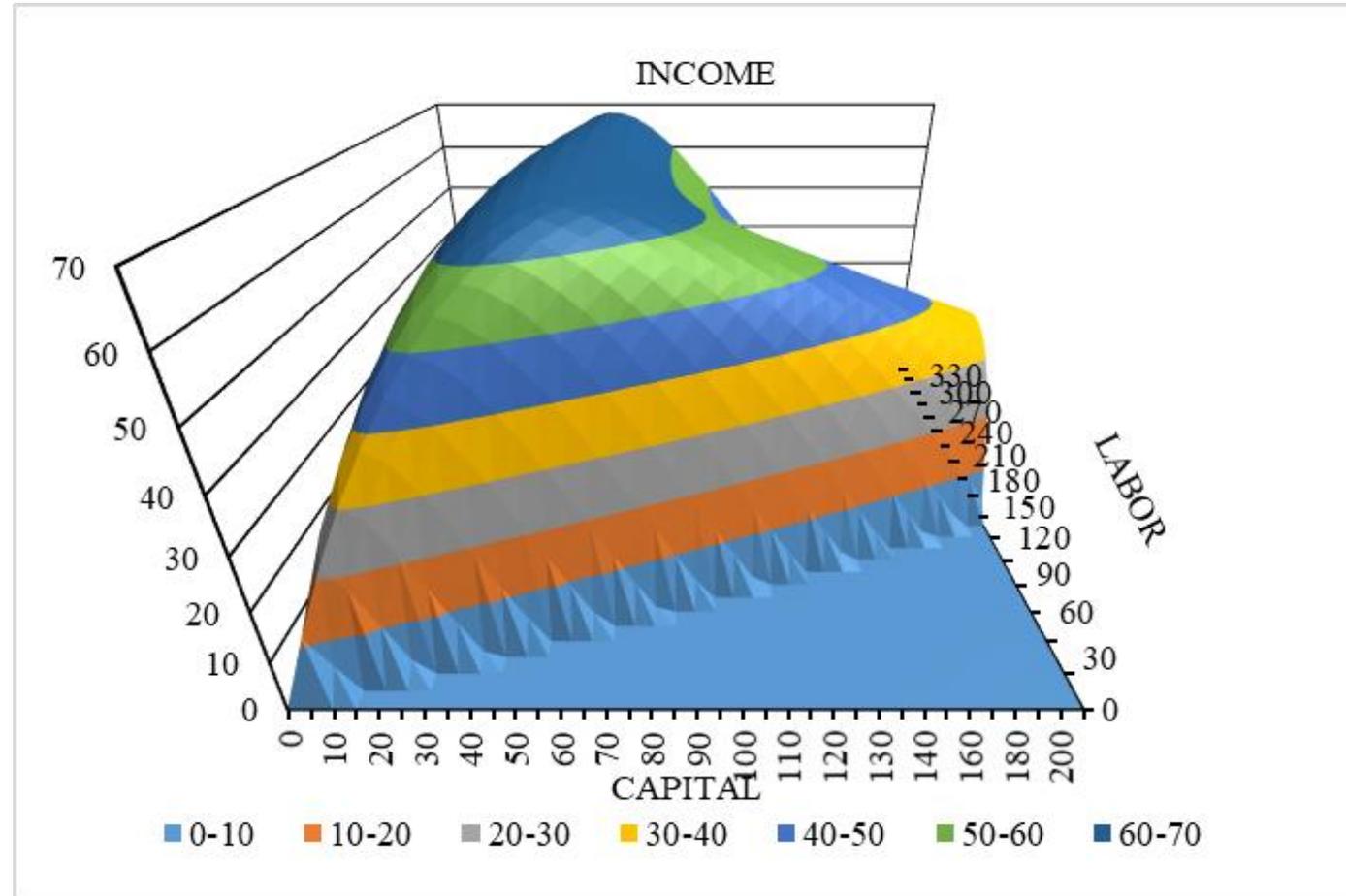
## Model of imperfect competition

Dependency of profit on capital and labor & The Pareto-optimal solutions in the criteria space: income and profit;



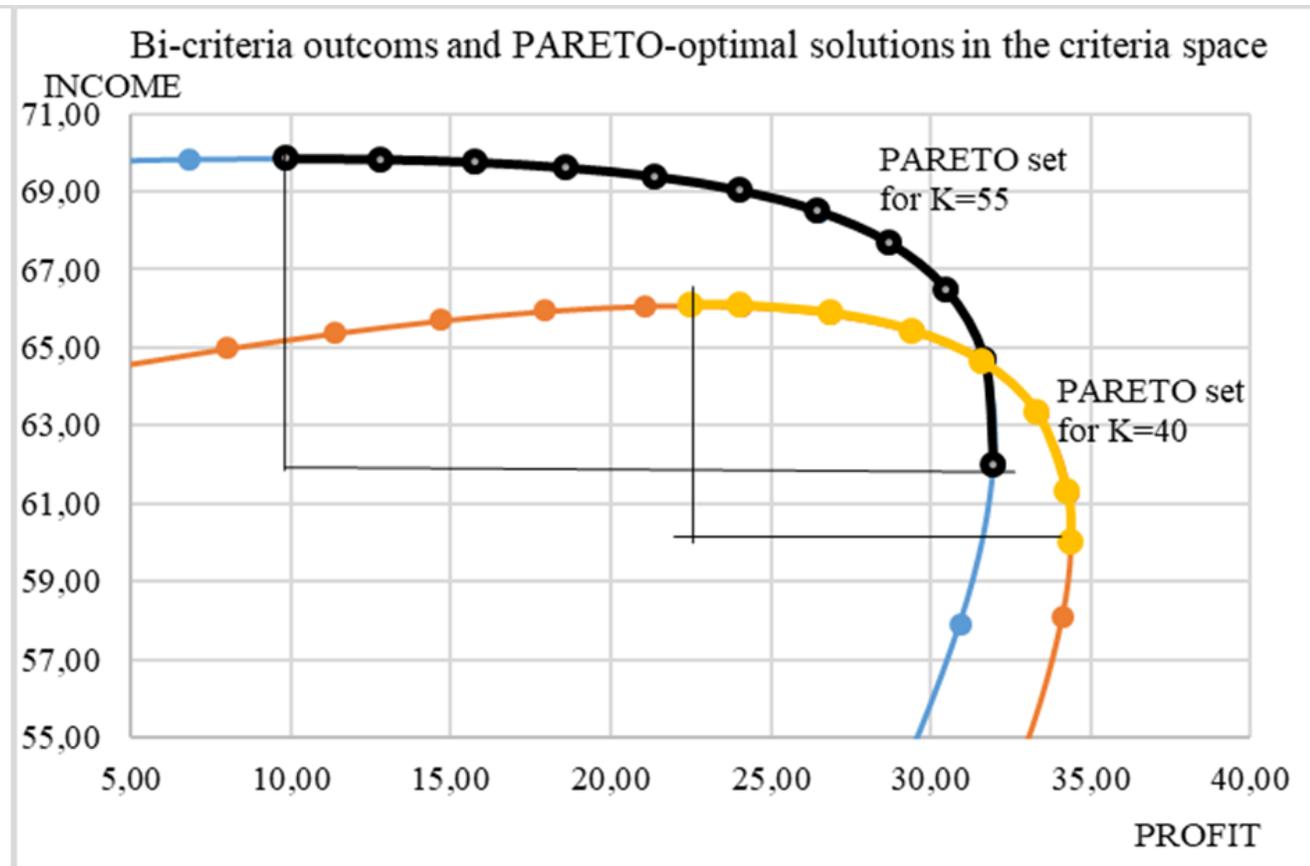
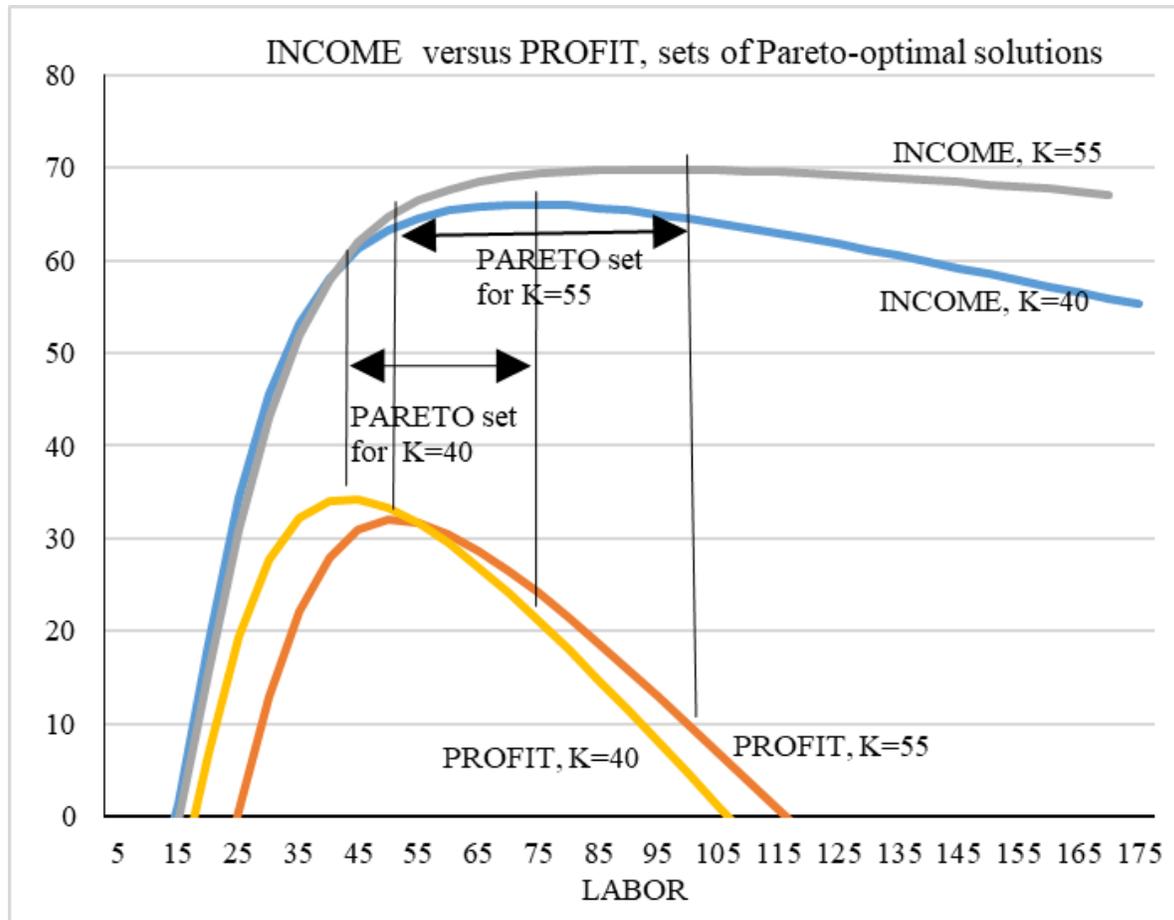
# Model of imperfect competition

The dependency of income on capital and labor;



# Model of imperfect competition

Income versus profit; dependency on capital for different values of labor;



## Remarks

Let representatives of two groups of interests negotiate decisions on the labor for given capital investments. The first group tries to obtain the maximal income possible, while the other is interested in maximization of profit.

The obtained results show that the criteria are contradictory. The groups negotiate the decision on the value of labor for a given value of capital assets.

The Pareto sets can be a ground of possible negotiations.

The negotiations should concern labor values as decisions which are defined by the Pareto set.

We can observe, see the last figures, that each decision out of the Pareto set can be improved for both criteria by a decision belonging to the set.

The Pareto sets narrow down sufficiently the domain of choice in the negotiations.

## Summary

- A model describing economic effects of an enterprise activity is presented. It is constructed using originally formulated production function, which satisfies the postulates proposed by R. Frisch.
- In the attached paper properties of the function are shown and analyzed for assumed model parameters. The theoretical considerations are illustrated by results of numerical experiments presenting productivities of the production factors in the short-term relationship between the production capacity and the input of labor, as well as contour lines of the long-term production function, the average costs and marginal cost as functions of the output.
- An exemplary bi-criteria optimization problem is formulated. The problem describes joint maximization of two criteria: income and profit, measuring objectives of the firm's activity with respect to capital and labor.
- The problem is analyzed in two market cases of the perfect and imperfect competition.
- The sets of Pareto-optimal solutions of the problem are derived.
- Conflicting internal interests in the firm may lead to different goals represented by such criteria as income and profit considered jointly. The Pareto set represents the set of alternatives which can be analyzed and discussed during possible negotiations. These solutions have the property that both criteria cannot be improved jointly. Therefore in this set a possible consensus can be looked for.

**Thank you for your attention!**